# Lecture Notes

# 05PC602
# POWER SYSTEM ANALYSIS

## VI Sem B.E (E & E)

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## 05PC602 POWER SYSTEM ANALYSIS

**Unit–I : Modelling of Power Systems Components**

Representation of power system components : Single phase solution of balanced three phase networks - One line diagram - Impedance or reactance diagram - Per unit system - Per unit impedance diagram - Complex power - representation of loads.

Review of symmetrical components - Transformation of voltage, current and impedance (conventional and power invariant transformations) - Phase shift in star- delta transformers - Sequence impedance of transmission lines - Sequence impedance and sequence network of power system components (synchronous machines, loads and transformer banks) - Construction of sequence networks of a power system.

**Unit–II : Bus Impedance and Admittance Matrices**

Development of network matrix from graph theory - Primitive impedance and admittance matrices - Bus admittance and bus impedance matrices – Properties - Formation of bus admittance matrix by inspection and analytical methods. Bus impedance matrix: Properties - Formation using building algorithm - addition of branch, link - removal of link, radial line - Parameter changes.

**Unit–III : Power Flow Analysis**

Sparsity - Different methods of storing sparse matrices - Triangular factorization of a sparse matrix and solution using the factors - Optimal ordering - Three typical schemes for optimal ordering - Implementation of the second method of Tinney and Walker. Power flow analysis - Bus classification - Development of power flow model - Power flow problem - Solution using Gauss Seidel method and Newton Raphson method - Application of sparsity based programming in Newton Raphson method - Fast decoupled load flow- comparison of the methods.

**Unit–IV : Fault Analysis**

Short circuit of a synchronous machine on no load and on load - Algorithm for symmetrical short circuit studies - Unsymmetrical fault analysis - Single line to ground fault, line to line fault, double line to ground fault ( with and without fault impedances ) using sequence bus impedance matrices - Phase shift due to star- delta transformers - Current limiting reactors - Fault computations for selection of circuit breakers.

**Unit–V : Short Circuit Study Based on Bus Admittance Matrix**

Phase and sequence admittance matrix representation for three phase, single line to ground, line to line and double line to ground faults (through fault impedances) - Computation of currents and voltages under faulted condition using phase and sequence fault admittance models - Sparsity based short circuit studies using factors of bus admittance matrix.

**Text Books**

1) Nagrath, I.J., Kothari. D.P., “Power System Engineering”, TMH, New Delhi; 2007.
2) Wadhwa, C.L., “Electric Power Systems”, Wiley Eastern, 2007.

**Reference Books**

1) Pai, M.A., “Computer Techniques in Power System Analysis”, TMH, 2007.
2) Stagg and El-Abiad, “Computer Methods in Power System Analysis”, McGraw Hill International, Student Edition, 1968.
3) Stevenson, W.D., “Element of Power System Analysis”, McGraw Hill, 1975.
4) Ashfaq Husain, “Electrical Power Systems”, CBS Publishers & Distributors, 1992.
5) Haadi Saadat, “Power System Analysis”, Tata McGraw Hill Edition, 2002.
6) Gupta, B.R., “Power System Analysis and Design, Third Edition”, A.H. Wheeler and Co Ltd., New Delhi, 1998.
7) Singh, L.P., “Advanced Power System Analysis and Dynamics, Fourth Edition, New Age International (P) Limited, Publishers, New Delhi, 2006.

UNIT-I

## Importance of Power System Studies:-

- A power system can be viewed as an interconnection of three main systems.

① Generator system - comprises synchronous machines, the exciter, the voltage regulator, the prime mover with governing mechanism etc.

② Transmission system - consists of transmission lines, transformers, protective relay apparatus, circuit breakers, static capacitors, shunt reactors, etc

③ Loads - modelled either as voltage dependent, current dependent or static impedance.

- Thus, todays' power systems are very complex and there are a number of decisions to be taken in a P.S both at the operational and at planning level.

- For example, the load dispatcher in a power system wants to judge the system behaviours and also the effectiveness of certain control strategies in the event of a particular disturbance. It is obviously not feasible to create such a disturbance on a real system; which in turn needs a very heavy emphasis on modelling and simulation techniques in digital computers.

- An appropriate simulation can provide the necessary data to sort out the merits of a particular control strategy.

- Similarly, in the planning level, the designer can even decide the location of future generation as well as the transmission network configuration well in advance. (5-10 years)

- The following studies are carried out for efficient design, operation and control of the power system.

1. Short ckt studies
2. Load flow studies
3. Transient stability
4. Dynamic stability
5. EHV transients
6. Relay coordination studies
7. Load forecasting
8. Maintenance scheduling
9. Economic allocation of generation
10. Unit commitment

- These studies ensures
  1. Proper planning
  2. Better economy
  3. Better quality
  4. Flexibility in operation.

**Functions of power system analysis**

- To monitor the voltage at various buses, real and reactive power flow between buses.
- To design the circuit breakers.
- To plan future expansion of the existing system
- To analyze the system under different fault conditions
- To study the ability of the system for small and large disturbances (Stability studies)

## COMPONENTS OF A POWER SYSTEM

1. Alternator

2. Power transformer

3. Transmission lines

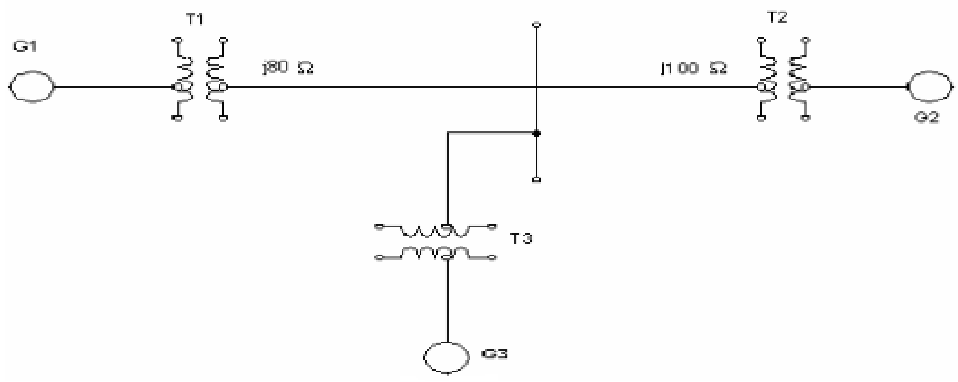
4. Substation transformer

5. Distribution transformer

6. Loads

## SINGLE LINE DIAGRAM

A single line diagram is diagrammatic representation of power system in which the components are represented by their symbols and interconnection between them are shown by a straight line(even-though the system is three phase system).The ratings and the impedance of the components are also marked on the single line diagram.




**Purpose of using single line diagram**

The purpose of the single line diagram is to supply in concise form of the significant information about the system.

**Per unit value.**

The per unit value of any quantity is defined as the ratio of the actual value of the any quantity to the base value of the same quantity as a decimal.

Per unit=Actual value / Base value

The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance rating of components of power system are expressed with reference to a common value called base value.

**Advantages of per unit system**

i. Per unit data representation yields valuable relative magnitude information.
ii. Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.
iii. The p.u systems are ideal for the computerized analysis and simulation of complex power system problems.
iv. Manufacturers usually specify the impedance values of equivalent in per unit of the equipments rating. If the any data is not available, it is easier to assume its per unit value than its numerical value.
v. The ohmic values of impedances are refereed to secondary is different from the value as referee to primary. However, if base values are selected properly, the p.u impedance is the same on the two sides of the transformer.
vi. The circuit laws are valid in p.u systems, and the power and voltages equations are simplified since the factors of √3 and 3 are eliminated.

**Change the base impedance from one set of base values to another set**

Let

Z=Actual impedance, Ω

$Z_b$=Base impedance, Ω

$$\text{Per unit impedance of a circuit element}=\frac{Z}{Z_b}=\frac{Z}{\frac{(kVb)^2}{MVA_b}}=\frac{Z\times MVA_b}{(kVb)^2} \qquad (1)$$

The eqn 1 show that the per unit impedance is directly proportional to base megavoltampere and inversely proportional to the square of the base voltage.

Using Eqn 1 we can derive an expression to convert the p.u impedance expressed in one base value ( old base) to another base (new base)

Let $kV_{b,old}$ and$MVA_{b,old}$ represents old base values and $kV_{b,new}$ and $MVA_{b,new}$ represent new base value

Let $Z_{p.u,old}$ = p.u. impedance of a circuit element calculated on old base

$Z_{p.u,new}$ = p.u. impedance of a circuit element calculated on new base

If old base values are used to compute the p.u.impedance of a circuit element, with impedance Z then eqn 1 can be written as

$$Z_{p.u,old} = \frac{Z \times MVA_{b,old}}{\left(kV_{b,old}\right)^2}$$

$$Z = Z_{p.u,old} \frac{\left(kV_{b,old}\right)^2}{MVA_{b,old}} \qquad (2)$$

If the new base values are used to compute the p.u. impedance of a circuit element with impedance Z, then eqn 1 can be written as

$$Z_{p.u,new} = \frac{Z \times MVA_{b,new}}{\left(kV_{b,new}\right)^2} \qquad (3)$$

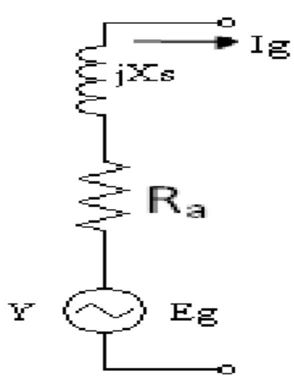
On substituting for Z from eqn 2 in eqn 3 we get

$$Z_{p.u,new} = Z_{p.u.old} \frac{\left(kV_{b,old}\right)^2}{MVA_{b,old}} \times \frac{MVA_{b,new}}{\left(kV_{b,new}\right)^2}$$

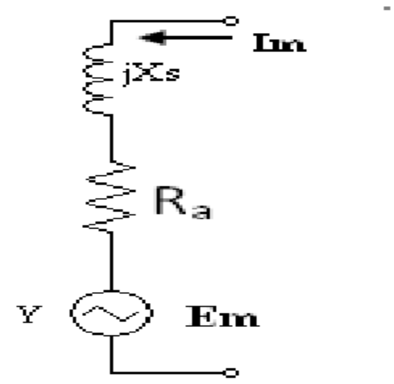
$$Z_{p.u,new} = Z_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right) \qquad (4)$$

The eqn 4 is used to convert the p.u.impedance expressed on one base value to another base

**MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR**

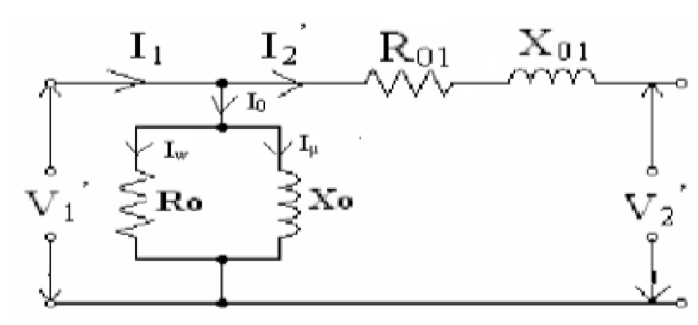



1Φ equivalent circuit of generator




1Φ equivalent circuit of synchronous motor

**MODELLING OF TRANSFORMER**

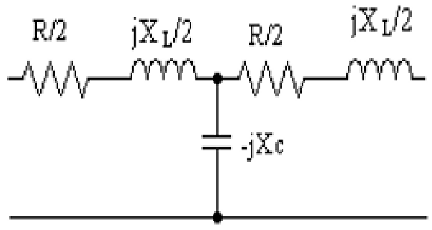


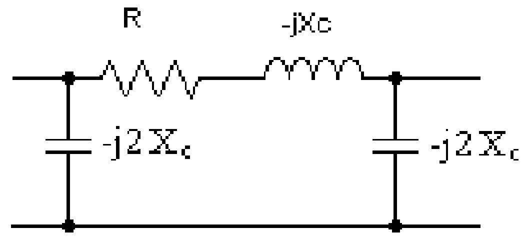

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1°

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1°

**MODELLING OF TRANSMISSION LINE**

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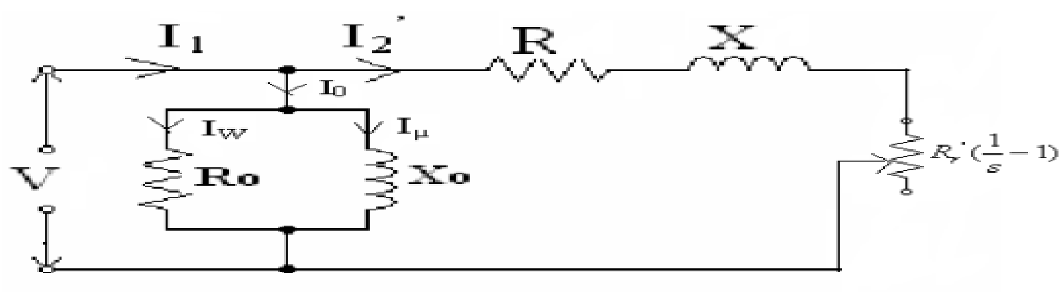
## MODELLING OF INDUCTION MOTOR




$R_r'\left(\frac{1}{s}-1\right)$ =Resistance representing load

$R = R_s + R_r'$ =Equivalent resistance referred to stator

$X = X_s + X_r'$ =Equivalent reactance referred to stator

### Impedance diagram & approximations made in impedance diagram

The impedance diagram is the equivalent circuit of power system in which the various components of power system are represented by their approximate or simplified equivalent circuits. The impedance diagram is used for load flow studies. Approximation: (i) The neutral reactances are neglected. (ii) The shunt branches in equivalent circuit of transformers are neglected.

### Reactance diagram & approximations made in reactance diagram

The reactance diagram is the simplified equivalent circuit of power system in which the various components of power system are represented by their reactances. The reactance diagram can be obtained from impedance diagram if all the resistive components are neglected. The reactance diagram is used for fault calculations.

Approximation:

(i) The neutral reactances are neglected.
(ii) The shunt branches in equivalent circuit of transformers are neglected.
(iii) The resistances are neglected.
(iv) All static loads are neglected.
(v) The capacitance of transmission lines are neglected

### PROCEDURE TO FORM REACTANCE DIAGRAM FROM SINGLE LINE DIAGRAM

1. Select a base power $kVA_b$ or $MVA_b$

2. Select a base voltage $kV_b$

3. The voltage conversion is achieved by means of transformer $kV_b$ on LT section

= $kV_b$ on HT section x LT voltage rating / HT voltage rating

4. When specified reactance of a component is in ohms

p.u reactance=Actual reactance/Base reactance

specified reactance of a component is in p.u

$$X_{p.u,new} = X_{p.u,old} * \frac{(kV_{b,old})^2}{(kV_{b,new})^2} * \frac{MVA_{b,new}}{MVA_{b,old}}$$

**EXAMPLE**

1. The single line diagram of an unloaded power system is shown in Fig 1.The generator transformer ratings are as follows.

G1=20 MVA, 11 kV, X''=25%

G2=30 MVA, 18 kV, X''=25%

G3=30 MVA, 20 kV, X''=21%

T1=25 MVA, 220/13.8 kV (Δ/Y), X=15%

T2=3 single phase units each rated 10 MVA, 127/18 kV(Y/Δ), X=15%

T3=15 MVA, 220/20 kV(Y/Δ), X=15%

Draw the reactance diagram using a base of 50 MVA and 11 kV on the generator1.

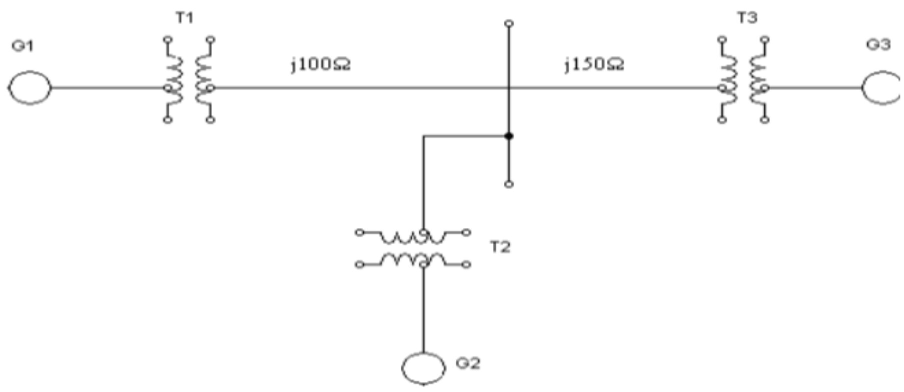



Fig 1

SOLUTION

Base megavoltampere,MVAb,new=50 MVA

Base kilovolt kVb,new=11 kV ( generator side)

***Reactance of Generator G***

$kV_{b,old}$=11 kV $\qquad$ $kV_{b,new}$=11 kV

$MVA_{b,old}$= 20 MVA $\qquad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.25p.u

$$\text{The new p.u. reactance of Generator G}=X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{20}\right) = j0.625p.u$$

side)

***Reactance of Transformer T1***

$kV_{b,old}$=11 kV $\qquad$ $kV_{b,new}$=11 kV

$MVA_{b,old}$= 25 MVA $\qquad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.15p.u

$$\text{The new p.u. reactance of Transformer T1}=X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right) = j0.3\ p.u$$

***Reactance of Transmission Line***

***It is connected to the HT side of the Transformer T1***

$$\text{Base kV on HT side of transformer } T1 = \text{Base kV on LT side} \times \frac{HT\ voltage\ rating}{LT\ voltage\ rating}$$

$$= 11 \times \frac{220}{11} = 220\ kV$$

*Actual Impedance* $X_{actual}$ = 100ohm

$$\text{Base impedance } X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{50} = 968\ ohm$$

$$\text{p.u reactance of } 100\ \Omega \text{ transmission line} = \frac{Actual\ \ Reactance\ ,ohm}{Base\ \ Reactance\ ,ohm} = \frac{100}{968} = j0.103\ p.u$$

$$\text{p.u reactance of } 150\ \Omega \text{ transmission line} = \frac{Actual\ \ Reactance\ ,ohm}{Base\ \ Reactance\ ,ohm} = \frac{150}{968} = j0.154\ p.u$$

**Reactance of Transformer T2**

$kV_{b,old} = 127 * \sqrt{3}\ kV = 220\ kV$ $\qquad$ $kV_{b,new} = 220\ kV$

$MVA_{b,old} = 10 * 3 = 30\ MVA$ $\qquad$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.15 p.u$

$$\text{The new p.u. reactance of Transformer } T2 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

**Reactance of Generator G2**

***It is connected to the LT side of the Transformer T2***

$$\text{Base kV on LT side of transformer } T2 = \text{Base kV on HT side} \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$$

$$= 220 \times \frac{18}{220} = 18\ kV$$

$kV_{b,old} = 18\ kV$ $\qquad$ $kV_{b,new} = 18\ kV$

$MVA_{b,old} = 30\ MVA$ $\qquad$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.25\ p.u$

$$\text{The new p.u. reactance of Generator } G2 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = j0.4167\ p.u$$

**Reactance of Transformer T3**

$kV_{b,old} = 20\ kV$ $\qquad$ $kV_{b,new} = 20\ kV$

$MVA_{b,old} = 20\ MVA$ $\qquad$ $MVA_{b,new} = 50\ MVA$

$X_{p.u,old} = 0.15 p.u$

$$\text{The new p.u. reactance of Transformer } T3 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

**Reactance of Generator G3**

***It is connected to the LT side of the Transformer T3***

$$\text{Base kV on LT side of transformer T 3} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{20}{220} = 20\ kV$$

$kV_{b,old}$=20 kV $\quad$ $kV_{b,new}$=20 kV

$MVA_{b,old}$= 30 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.21 p.u

$$\text{The new p.u. reactance of Generator G 3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.21 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.35\ p.u$$

**Example**

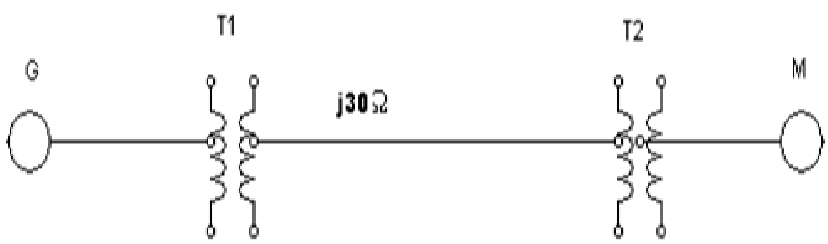
2) Draw the reactance diagram for the power system shown in fig .Use a base of50 MVA , 230 kV in 30 Ω line. The ratings of the generator, motor and transformers are

Generator = 20 MVA, 20 kV, X=20%

Motor = 35 MVA, 13.2 kV, X=25%

T1 = 25 MVA, 18/230 kV (Y/Y), X=10%

T2 = 45 MVA, 230/13.8 kV (Y/Δ), X=15%




**Solution**

Base megavoltampere,MVAb,new=50 MVA

Base kilovolt kVb,new=230 kV ( Transmission line side)

***FORMULA***

$$\text{The new p.u. reactance } X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

*Reactance of Generator G*

*It is connected to the LT side of the T1 transformer*

$$\text{Base kV on LT side of transformer T 1} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 230 \times \frac{18}{230} = 18\ kV$$

$kV_{b,old}$=20 kV $\quad$ $kV_{b,new}$=18 kV

$MVA_{b,old}$= 20 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.2p.u

$$\text{The new p.u. reactance of Generator G} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{20}{18}\right)^2 \times \left(\frac{50}{20}\right) = j0.617\ p.u$$

*Reactance of Transformer T1*

$kV_{b,old}$=18 kV $\quad$ $kV_{b,new}$=18 kV

$MVA_{b,old}$= 25 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.1p.u

$$\text{The new p.u. reactance of Transformer T1}=X_{pu,old}\times\left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2\times\left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.1\times\left(\frac{18}{18}\right)^2\times\left(\frac{50}{25}\right)=j0.2\ p.u$$

*Reactance of Transmission Line*

*It is connected to the HT side of the Transformer T1*

*Actual Impedance* $X_{actual}= j30$ *ohm*

$$\text{Base impedance } X_{base}=\frac{(kV_{b,new})^2}{MVA_{b,new}}=\frac{230^2}{50}=1058\ ohm$$

$$\text{p.u reactance of } j30\ \Omega \text{ transmission line}=\frac{Actual\ \ Reactance\ ,ohm}{Base\ \ Reactance\ ,ohm}=\frac{j30}{1058}=j0.028\ p.u$$

**Reactance of Transformer T2**

$kV_{b,old}$=230 kV $\quad$ $kV_{b,new}$=230 kV

$MVA_{b,old}$= 45 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.15p.u

$$\text{The new p.u. reactance of Transformer T2}=X_{pu,old}\times\left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2\times\left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.15\times\left(\frac{230}{230}\right)^2\times\left(\frac{50}{45}\right)=j0.166\ p.u$$

**Reactance of Motor M2**

***It is connected to the LT side of the Transformer T2***

$$\text{Base kV on LT side of transformer T 2}=\text{Base kV on HT side}\times\frac{LT\ voltage\ \ rating}{HT\ voltage\ \ rating}$$

$$=230\times\frac{13.8}{230}=13.8\ kV$$

$kV_{b,old}$=13.2 kV $\quad$ $kV_{b,new}$=13.8 kV

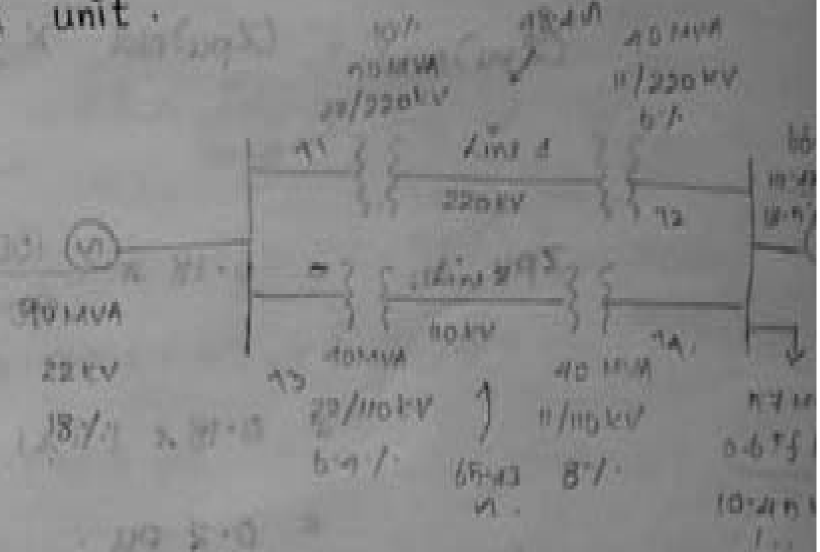
$MVA_{b,old}$= 35 MVA $\quad$ $MVA_{b,new}$=50 MVA

$X_{p.u,old}$=0.25 p.u

$$\text{The new p.u. reactance of Generator G 2}=X_{pu,old}\times\left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2\times\left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$=0.25\times\left(\frac{13.2}{13.8}\right)^2\times\left(\frac{50}{35}\right)=j0.326\ p.u$$

1). Draw the reactance diagram using a common base of ~~30 MVA~~ 100 MVA and 22 KV on the generated side. All the impedances including the load impedance are marked in per unit.




Given:

$$MVA_B = 100\ MVA.$$
$$KV_B = 22\ KV.$$

Soln:

The base voltage on the High voltage side of Transformer $T_1$ is $22 \times \frac{220}{22} = 220\ KV = V$

The base voltage on the low voltage side of transformer $T_2$ is $220 \times \frac{11}{220} = 11\ KV. = V_{T_2}$

The base voltage on the HV side of $T_3$ is $22 \times \frac{110}{22} = 110\ KV = V_{T_3}$

The base voltage on the LV side of $T_4$ is $110 \times \frac{11}{110} = 11\ KV. = V_{T_4}$

The generator and transformer per unit reactances on 100 MVA B can be calculated using

$$(Z_{pu})_{new} = (Z_{pu})_{old} \times \frac{MVA_B^{new}}{MVA_B^{old}} \times \left(\frac{KV_B^{old}}{KV_B^{new}}\right)^2.$$

$$Z_{pu}\ G_1 = 0.18 \times \frac{100}{90} \times \left(\frac{22}{22}\right)^2$$

$$= 0.18 \times 1.11 \times 1$$

$$= 0.2 \text{ pu}.$$

$$Z_{pu}\ T_1 = 0.1 \times \frac{100}{50} \times \left(\frac{22}{22}\right)^2$$

$$= 0.1 \times 2$$

$$= 0.2 \text{ pu}.$$

$$Z_{pu}\ T_2 = 0.06 \times \frac{100}{40} \times \left(\frac{11}{11}\right)^2$$

$$= 0.06 \times 2.5$$

$$= 0.15 \text{ pu}.$$

$$Z_{pu}\ T_3 = 0.064 \times \frac{100}{40} \times \left(\frac{110}{110}\right)^2$$

$$= 0.064 \times 2.5$$

$$= 0.16 \text{ pu}$$

$$Z_{pu}\ T_4 = 0.08 \times \frac{100}{40} \times \left(\frac{11}{11}\right)^2$$

$$= 0.08 \times 2.5$$

$$= 0.2 \text{ pu.}$$

$$Z_{pu}\ M = 0.185 \times \frac{100}{66.5} \times \left(\frac{10.45}{11}\right)^2$$

$$= 0.185 \times 1.503 \times (0.95)^2$$

$$= 0.185 \times 1.503 \times 0.9025$$

$$= 0.2509 \text{ pu.}$$

$$\text{per unit} = \frac{\text{Actual}}{\text{Base}}$$

$$Z_{TL_1} = \cancel{48.8}\ \frac{48.4}{\left[\frac{(220)^2}{100}\right]}$$

$$= \frac{48.4}{\left(\frac{48400}{100}\right)}$$

$$= 0.1 \text{ pu}$$

$$Z_{TL_2} = \frac{65.43}{\cancel{48.4}\left[\frac{(110)^2}{100}\right]}$$

$$= \frac{65.43}{\left(\frac{12100}{100}\right)} = 0.54 \text{ pu}$$

Load ⇒ Zpu.

$$\text{Actual } Z_{load} = \frac{(kV)^2}{MVA} = \frac{(10.5)^2}{57\angle -53.13} = 1.9158\angle 53.13$$

$$\cos\theta = 0.6$$

$$\theta = \cos^{-1}(0.6)$$

$$= 53.13^\circ$$

$$\text{pu value} = \frac{Z_{Act}}{Z_{Base}}$$

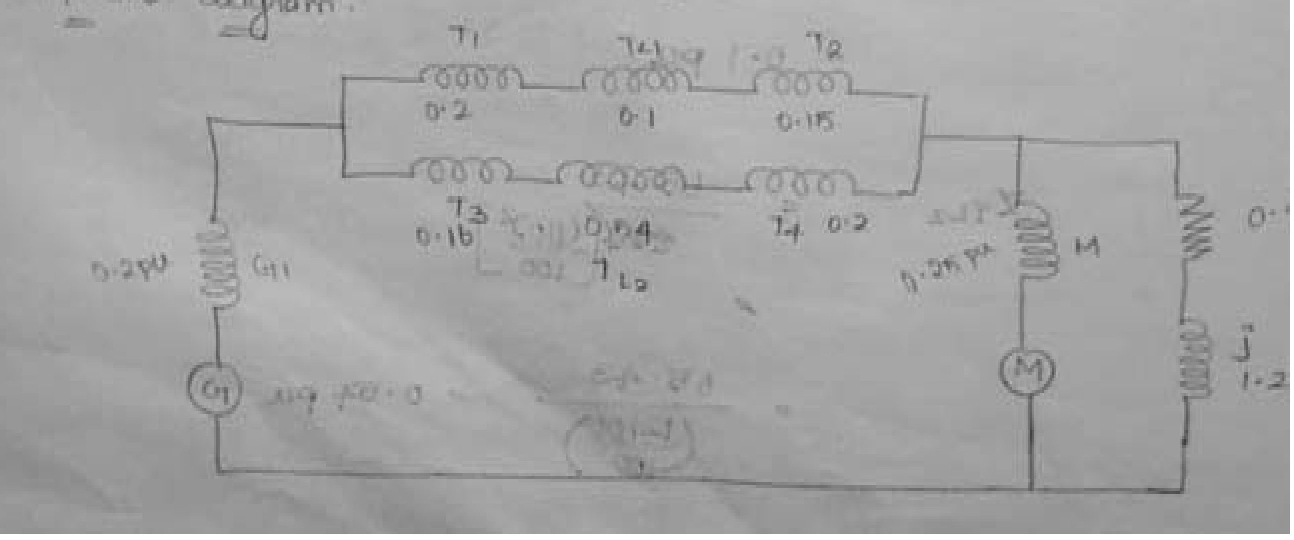
$$= \frac{1.9158\angle 53.13}{(11)^2/100}$$

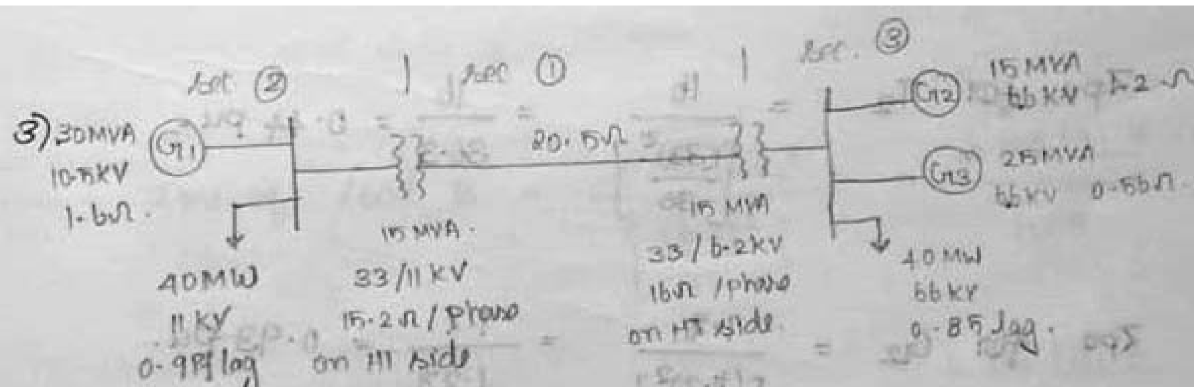
$$= \frac{1.9158\angle 53.13}{1.21}$$

$$= 1.5833\angle 53.13$$

$$= 0.9499 + j1.26$$

Impedance diagram




Obtain the per unit impedance diagram of the power system shown in figure. Assume a base of 30 MVA, 33 kV on the transmission line.

Soln: Base values of section 1 is 30 MVA and 33 kV

Base values of Section 2 is 30 MVA and 11 kV

Base values of section 3 is 30 MVA and 6.2 kV.

$$Z_{pu} \text{ for } G_1 = \frac{Z_{act}}{Z_{base}} = \frac{1.6}{\frac{(11)^2}{30}} = \frac{1.6}{4.03} = 0.39 \text{ pu.}$$

$$Z_{pu} \text{ for } T_1 = \frac{Z_{act}}{Z_{base}} = \frac{15.2}{\left[\frac{(33)^2}{30}\right]} = \frac{15.2}{36.3} = 0.41 \text{ pu.}$$

$$Z_{pu} \text{ for Transmission line} = \frac{20.5}{\left[\frac{(33)^2}{30}\right]} = \frac{20.5}{36.3} = 0.56 \text{ pu.}$$

$$Z_{pu} \text{ for } T_2 = \frac{16}{\left[\frac{(33)^2}{30}\right]} = \frac{16}{36.3} = 0.44 \text{ pu.}$$

$$Z_{pu} \text{ for } G_2 = \frac{1.2}{\left[\frac{(6.2)^2}{30}\right]} = \frac{1.2}{1.28} = 0.93 \text{ pu.}$$

$$Z_{pu} \text{ for } G_3 = \frac{0.56}{\left[\frac{(6.2)^2}{30}\right]} = \frac{0.56}{1.28} = 0.43 \text{ pu}$$

$$\text{Load impedance} = \frac{(kV)^2}{MVA} = \frac{(11)^2}{40\angle -\cos^{-1}(0.9)}$$

$$= \frac{121}{40\angle -25.8}$$

$$= 3.02\angle 25.8 \ \Omega.$$

$$Z_{pu} \text{ of Load A} = \frac{3.02}{\left[\frac{(11)^2}{30}\right]} = \frac{3.02\angle 25.8}{4.03}$$

$$= 0.74\angle 25.8$$

$$= 0.66 + 0.32j.$$

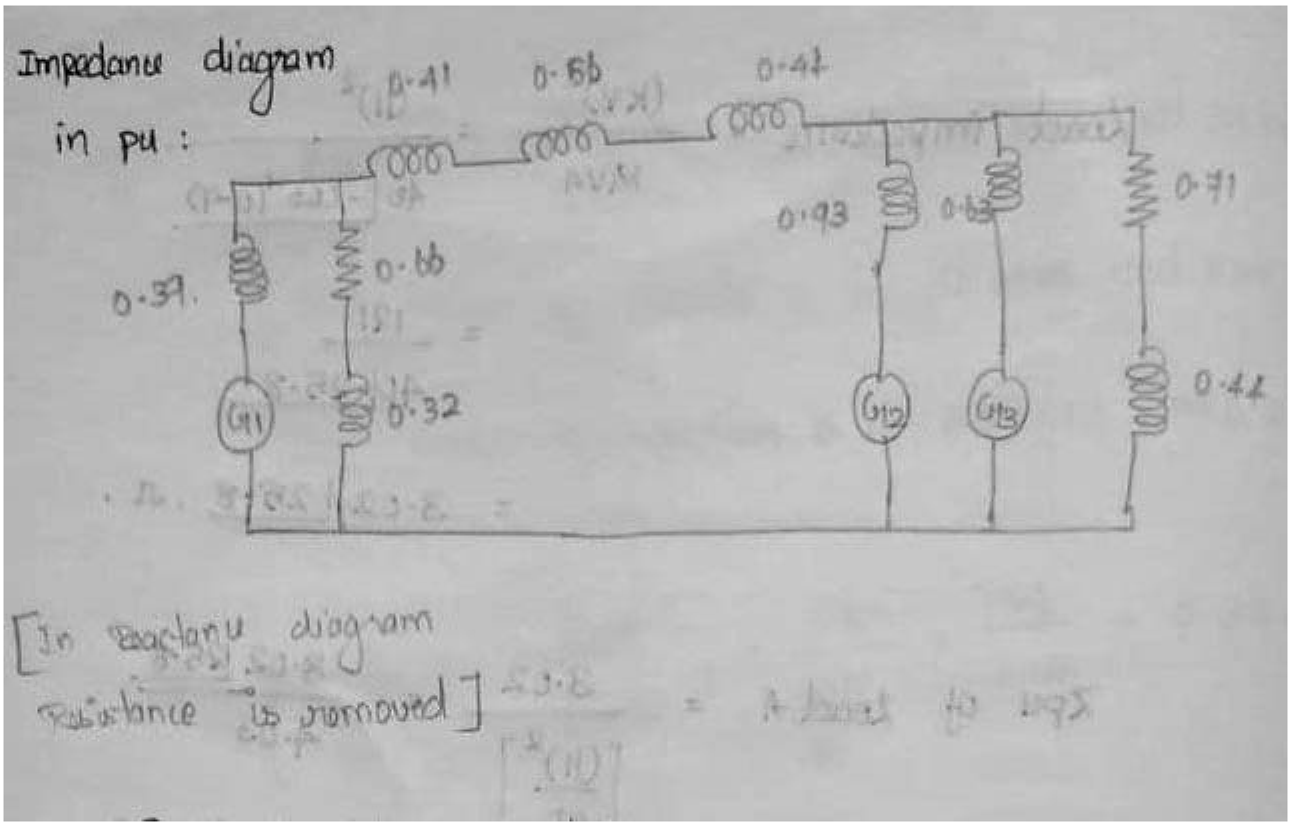
$$Z_{act} \text{ of Load B} = \frac{(6.6)^2}{40\angle -\cos^{-1}(0.85)}$$

$$= \frac{43.56}{40\angle -31.7} = 1.08\angle 31.7$$

$$Z_{pu} \text{ of Load B} = \frac{1.08\angle 31.7}{\left[\frac{(6.2)^2}{30}\right]} = \frac{1.08\angle 31.7}{1.28}$$

$$= 0.84\angle 31.7$$

$$= 0.71 + j0.44.$$




Representation of load :

complex power.

$S = P + jQ = VI^*$

$S^* = P - jQ = V^*I$

* constant ~~load~~ power $\Rightarrow$ Real $\Rightarrow$ MW
  Reactive $\Rightarrow$ MVAR } lower frequency.

* constant current $\Rightarrow I = \dfrac{P-jQ}{V^*}$.
  (for short circuit current)

* constant impedance $\Rightarrow Z = \dfrac{V}{I} = \dfrac{V \cdot V^*}{P-jQ} = \dfrac{|V|^2}{P-jQ}$.

**Symmetrical Components**

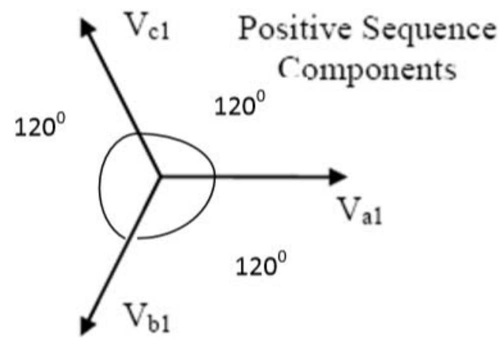
An unbalanced system of N related vectors can be resolved into N systems of balanced vectors. The N – sets of balanced vectors are called symmetrical components. Each set consists of N – vectors which are equal in length and having equal phase angles between adjacent vectors.

**Sequence Impedance and Sequence Network**

The sequence impedances are impedances offered by the devices or components for the like sequence component of the current .The single phase equivalent circuit of a power system consisting of impedances to the current of any one sequence only is called sequence network.

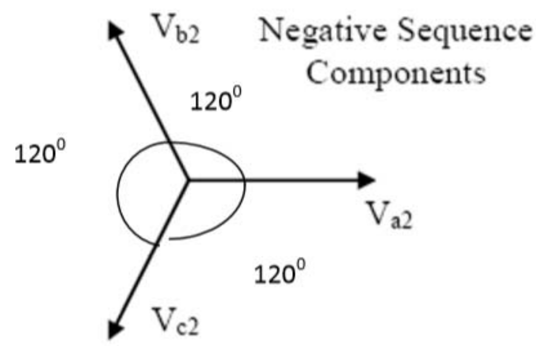
**Positive Sequence Components**

The positive sequence components are equal in magnitude and displayed from each other by 120o with the same sequence as the original phases. The positive sequence currents and voltages follow the same cycle order of the original source. In the case of typical counter clockwise rotation electrical system, the positive sequence phasor are shown in Fig . The same case applies for the positive current phasors. This sequence is also called the “abc” sequence and usually denoted by the symbol “+” or “1”



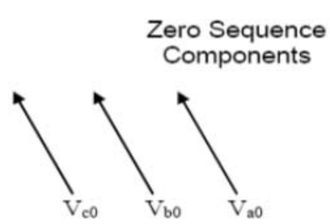

**Negative Sequence Components**

This sequence has components that are also equal in magnitude and displayed from each other by 120o similar to the positive sequence components. However, it has an opposite phase sequence from the original system. The negative sequence is identified as the "acb" sequence and usually denoted by the symbol "-" or "2" [9].The phasors of this sequence are shown in Fig  where the phasors rotate anti- clockwise. This sequence occurs only in case of an unsymmetrical fault in addition to the positive sequence components,




**Zero Sequence Components**

In this sequence, its components consist of three phasors which are equal in magnitude as before but with a zero displacement. The phasor components are in phase with each other. This is illustrated in Fig . Under an asymmetrical fault condition, this sequence symbolizes the residual electricity in the system in terms of voltages and currents where a ground or a fourth wire exists. It happens when ground currents return to the power system through any grounding point in the electrical system. In this type of faults, the positive and the negative components are also present. This sequence is known by the symbol "0" .



Symmetrical components :

voltage same ⇐ positive sequence . ⇒ 1 ⇒  ⇒ equal in magnitude and angle

no same ⇐ Negative sequence ⇒ 2 ⇒  ⇐ equal in magnitude and angle but opposite direction

no source ⇐ Zero sequence . ⇒ 0

⇓

 ⇐ without any phase sequence

unbalanced component = Sum of all balanced sequence component.

ie, $\vec{V}_a = \vec{V}_{a0} + \vec{V}_{a1} + \vec{V}_{a2}$

$\vec{V}_b = \vec{V}_{b0} + \vec{V}_{b1} + \vec{V}_{b2}$ .

$\vec{V}_c = \vec{V}_{c0} + \vec{V}_{c1} + \vec{V}_{c2}$ .

Vector operator :

$a = 1\angle 120^\circ$ = shifts angle $120^\circ$ by counter clockwise $= -0.5 + j0.866$

$a^2 = 1\angle 240^\circ = -0.5 - j0.866$

$a^3 = 1\angle 360^\circ = 1.$

$$\boxed{1 + a + a^2 = 0}$$

If we take $V_a$ as reference, we get

⇒ $\vec{V}_b = \vec{V}_{a0} + a^2\vec{V}_{a1} + a\vec{V}_{a2}$

$\vec{V}_c = \vec{V}_{a0} + a\vec{V}_{a1} + a^2\vec{V}_{a2}$

Thus,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\Rightarrow \quad [V_{ph}] = [A][V_{seq}] \quad \text{——} \; ①$$

$$\therefore \quad [V_{seq}] = [A]^{-1}[V_{ph}] \quad \text{——} \; ②$$

Hence, $$[A^{-1}] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Now ② becomes,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a0} = \frac{1}{3}\left[V_a + V_b + V_c\right]$$

$\cancel{V_{a0}}$

$$V_{a1} = \frac{1}{3}\left[V_a + aV_b + a^2V_c\right]$$

$$V_{a2} = \frac{1}{3}\left[V_a + a^2V_b + aV_c\right].$$

$|||^{ly}$, $I_{ph} = A\, I_{seq}$.

$I_{seq} = A^{-1} I_{ph}$.

power Invarient :

$$S = [V_{ph}]^T [I_{ph}]^*$$

$$= [A\, V_{Beq}]^T [A\, I_{Beq}]^*$$

$$\therefore S = V_{Beq}^T A^T \cdot A^* I_{Beq}^* \quad \text{——— (3)}$$

$$A^T A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Hence
$1+a+a^2=0$
$a^4=a$
$a^3=1$

$$= \begin{bmatrix} 1+1+1 & 1+a+a^2 & 1+a^2+a \\ 1+a^2+a & 1+a^3+a^3 & 1+a^4+a^2 \\ 1+a+a^2 & 1+a^2+a^4 & 1+a^3+a^3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3[U].$$

Now, (5) $\Rightarrow$ $S = V_{Beq}^T \cdot 3[U]\, I_{Beq}^*$

$$\boxed{\therefore S = 3\, V_{Beq}^T I_{Beq}^*}$$

**EXAMPLE**

1. The symmetrical components of a phase –a voltage in a 3-phase unbalanced system are $V_{a0} = 10\angle 180^0$ V, $V_{a1} = 50\angle 0^0$ V and $V_{a2} = 20\angle 90^0$ V.

   Determine the phase voltages Va ,Vb and Vc

The phase voltages of $V_a, V_b\, and\, V_c$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

$$V_{a0} = 10\angle 180^0 = -10 + j0 \ \text{V}$$

$$V_{a1} = 50\angle 0^0 = 50 + j0 \ \text{V}$$

$$V_{a2} = 20\angle 90^0 = 0 + j20 \ \text{V}$$

a=$1\angle 120^0$ $a^2$= $1\angle 240^0$

$$a^2 V_{a1} = 1\angle 240^0 \times 50\angle 0^0 = 50\angle 240^0 = -25 - \text{j}43.30$$

$$a V_{a1} = 1\angle 120^0 \times 50\angle 0^0 = 50\angle 120^0 = -25 + \text{j}43.30$$

$$a^2 V_{a2} = 1\angle 240^0 \times 20\angle 90^0 = 20\angle 233 = 17.32 - \text{j}10$$

$$a V_{a2} = 1\angle 120^0 \times 20\angle 90^0 = 20\angle 210^0 = -17.32 - \text{j}10$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = (-10 + j0) + (50 + j0) + (0 + j20) = 40 + j20 = 44.72\angle 27^0\ V$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = (-10 + j0) + (-25 - \text{j}43.30) + (-17.32 - \text{j}10) = -52.32 - j53.90$$
$$= 74.69\angle -134^0 \text{ V}$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = (-25 - \text{j}43.30) + (-25 + \text{j}43.30) + 17.32 - \text{j}10 = -17.68 + \text{j}33.3$$

$$= 37.70\ \angle -118^0\ V$$

## THREE-SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

Positive sequence currents give rise to only positive sequence voltages, the negative sequence currents give rise to only negative sequence voltages and zero sequence currents give rise to only zero sequence voltages, hence each network can be regarded as flowing within in its own network through impedances of its own sequence only.

In any part of the circuit, the voltage drop caused by current of a certain sequence depends on the impedance of that part of the circuit to current of that sequence.

The impedance of any section of a balanced network to current of one sequence may be different from impedance to current of another sequence.

The impedance of a circuit when positive sequence currents are flowing is called impedance, When only negative sequence currents are flowing the impedance is termed as negative sequence impedance. With only zero sequence currents flowing the impedance is termed as zero sequence impedance.

The analysis of unsymmetrical faults in power systems is carried out by finding the symmetrical components of the unbalanced currents.

Since each sequence current causes a voltage drop of that sequence only, each sequence current can be considered to flow in an independent network composed of impedances to current of that sequence only.

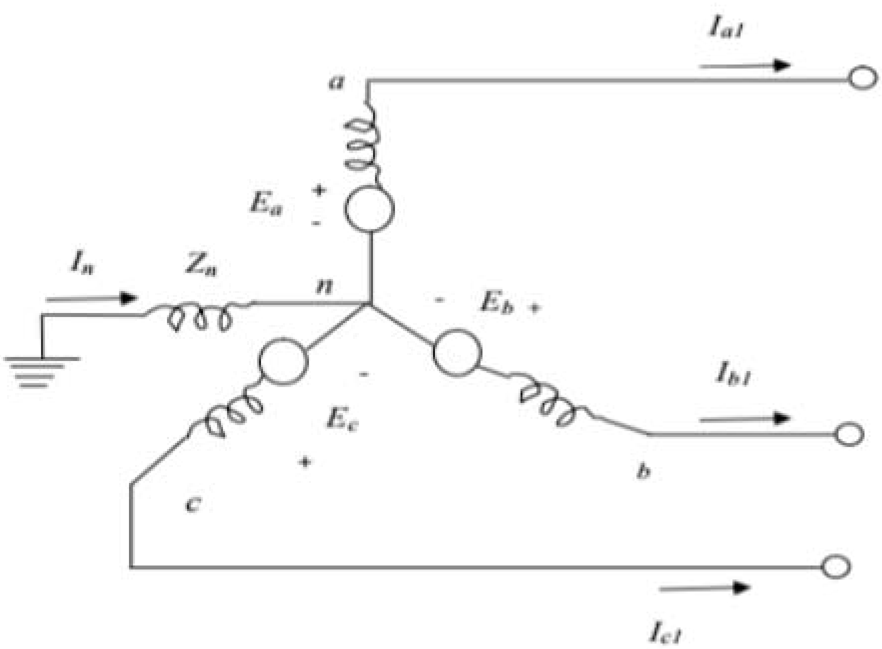
The single phase equivalent circuit composed of the impedances to current of any one sequence only is

called the sequence network of that particular sequence. The sequence networks contain the generated emfs and impedances of like sequence. Therefore for every power system we can form three- sequence network s. These sequence networks, carrying current Ia1, Ia2 and Ia0 are then inter-connected to represent the different fault conditions.

**SEQUENCE NETWORKS OF SYNCHRONOUS MACHINES**

An unloaded synchronous machine having its neutral earthed through impedance, Zn, is shown in fig. below. A fault at its terminals causes currents Ia, Ib and Ic to flow in the lines. If fault involves earth, a current In flows into the neutral from the earth. This current flows through the

neutral impedance Zn. Thus depending on the type of fault, one or more of the line currents may be zero. Thus depending on the type of fault, one or more of the line currents may be zero.




**POSITIVE SEQUENCE NETWORK**

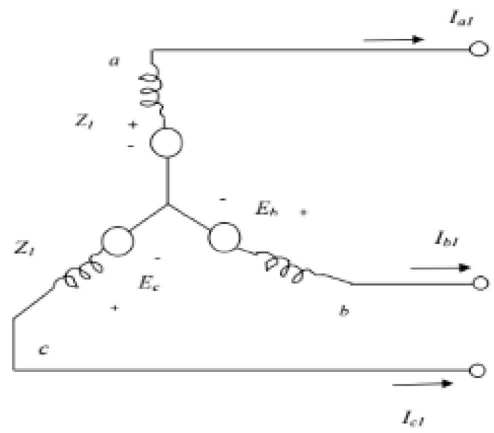
The generated voltages of a synchronous machine are of positive sequence only since the windings of a synchronous machine are symmetrical.

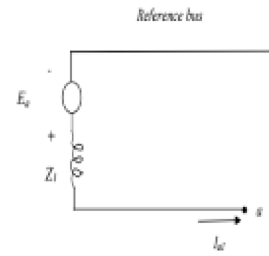
The positive sequence network consists of an emf equal to no load terminal voltages and is in series with the positive sequence impedance Z1 of the machine. Fig.2 (b) and fig.2(c) shows the paths for positive sequence currents and positive sequence network respectively on a single phase basis in the synchronous machine.

The neutral impedance Zn does not appear in the circuit because the phasor sum of $Ia_1$, $Ib_1$ and $Ic_1$ is zero and no positive sequence current can flow through Zn. Since its a balanced circuit, the positive sequence N The reference bus for the positive sequence network is the neutral of the generator. The positive sequence impedance $Z_1$ consists of winding resistance and direct axis reactance. The reactance is the sub-transient reactance X"d or transient reactance X'd or synchronous reactance Xd depending on whether sub-transient, transient or steady state conditions are being studied. From fig.2 (b) ,

the positive sequence voltage of terminal a with respect to the reference bus is given by:

$Va_1 = Ea - Z_1 Ia_1$






**NEGATIVE SEQUENCE NETWORK**

A synchronous machine does not generate any negative sequence voltage. The flow of negative sequence currents in the stator windings creates an mmf which rotates at synchronous speed in a direction opposite to the direction of rotor, i.e., at twice the synchronous speed with respect to rotor.

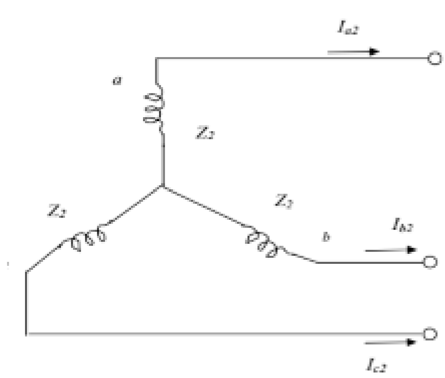
Thus the negative sequence mmf alternates past the direct and quadrature axis and sets up a varying armature reaction effect. Thus, the negative sequence reactance is taken as the average of direct axis and quadrature axis sub-transient reactance, i.e.,

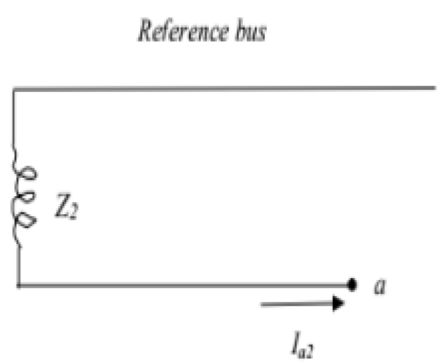
$X_2 = 0.5\ ( X''d + X''q )$.

It not necessary to consider any time variation of X2 during transient conditions because there is no normal constant armature reaction to be effected. For more accurate calculations, the negative sequence resistance should be considered to account for power dissipated in the rotor poles or damper winding by double supply frequency induced currents. The fig.below shows the negative sequence currents paths and the negative sequence network respectively on a single phase basis of a synchronous machine. The reference bus for the negative sequence network is the neutral of the machine.

Thus, the negative sequence voltage of terminal a with respect to the reference bus is given by:

$Va_2 = -Z_2Ia_2$



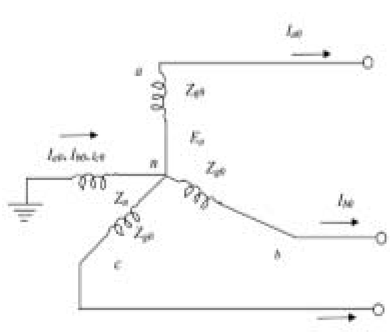


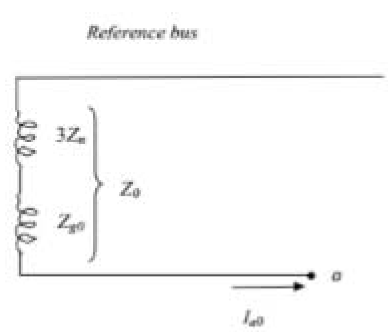

**ZERO SEQUENCE NETWORK**

No zero sequence voltage is induced in a synchronous machine. The flow of zero sequence currents in the stator windings produces three mmf which are in time phase. If each phase winding produced a sinusoidal space mmf, then with the rotor removed, the flux at a point on the axis of the stator due to zero sequence current would be zero at every instant.

When the flux in the air gap or the leakage flux around slots or end connections is considered, no point in these regions is equidistant from all the three –phase windings of the stator.

The mmf produced by a phase winding departs from a sine wave, by amounts which depend upon the arrangement of the winding.





## 3.9 Sequence Impedances of Transmission Lines

Consider a transmission system where the self impedance of each phase be represented by $X_s$ and the mutual impedance between any of the two phases be represented by $X_m$

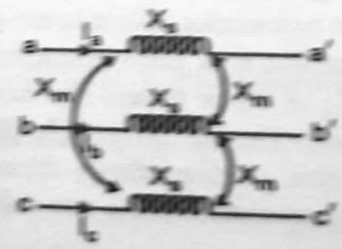


Fig. 3.14 A transmission system.

Let

$$V'_{aa} \rightarrow \text{Voltage in phase } a \rightarrow V_a$$

$$V'_{bb} \rightarrow \text{Voltage in phase } b \rightarrow V_b$$

$$V'_{cc} \rightarrow \text{Voltage in phase } c \rightarrow V_c$$

If $I_a$, $I_b$ and $I_c$ represent the phase currents, then

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

This is of the form

$$V^{abc} = Z^{abc} I^{abc}$$

Converting it to symmetrical components, we get

$$V^{012} = A^{-1} Z^{abc} A I^{012}$$

$$A^{-1} Z_{abc} A = A^{-1} j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= j\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} X_s+2X_m & X_s+a^2X_m+aX_m & X_s+aX_m+a^2X_m \\ X_s+2X_m & X_m+a^2X_s+aX_m & X_m+aX_s+a^2X_m \\ X_s+2X_m & X_m+a^2X_m+aX_s & X_m+aX_m+a^2X_s \end{bmatrix}$$

$$= \begin{bmatrix} j(X_s+2X_m) & 0 & 0 \\ 0 & j(X_s-X_m) & 0 \\ 0 & 0 & j(X_s-X_m) \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a1} \end{bmatrix} = j \begin{bmatrix} X_s+2X_m & 0 & 0 \\ 0 & X_s-X_m & 0 \\ 0 & 0 & X_s-X_m \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

We conclude that for a transmission line

1. Positive and negative sequence impedances are equal.
2. Zero sequence impedance is approximately 2.5 times that of positive or negative sequence impedance in the case of single circuit lines. For double circuit lines, the order will be more.

In all our power system problems while drawing sequence networks, we assume all the three sequence impedances of a transmission line as equal to the leakage impedance unless specifically mentioned.

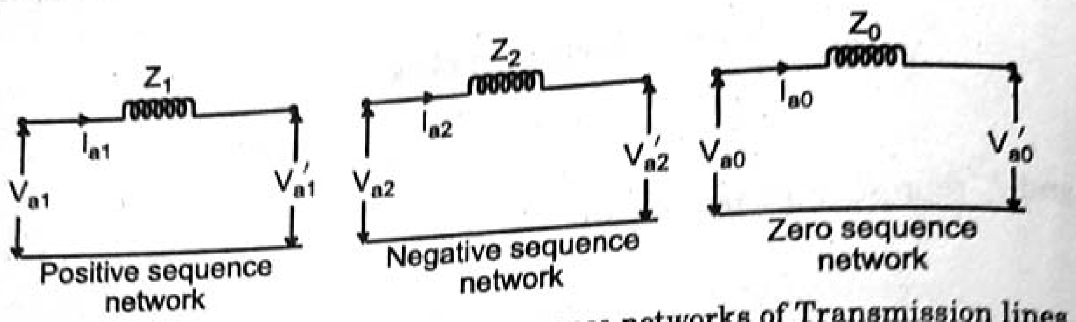



**Fig. 3.15** Positive, Negative and Zero sequence networks of Transmission lines.

## 3.10 Sequence Network of Transformer

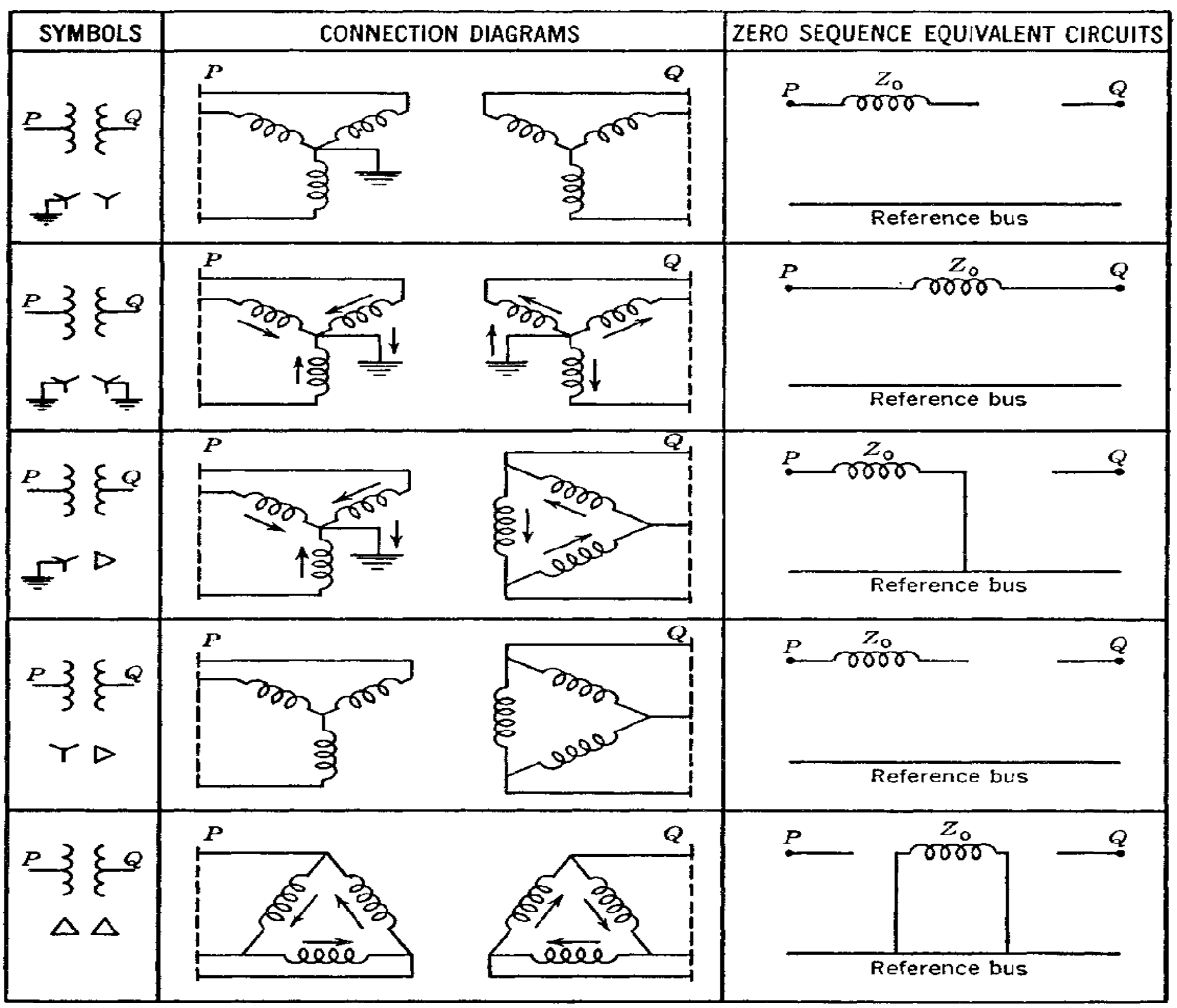
The positive and negative sequence network of a three phase transformer is as per our usual representation by leakage impedances.

$$Z_1 = Z_2 = Z_{\text{leakage}}.$$

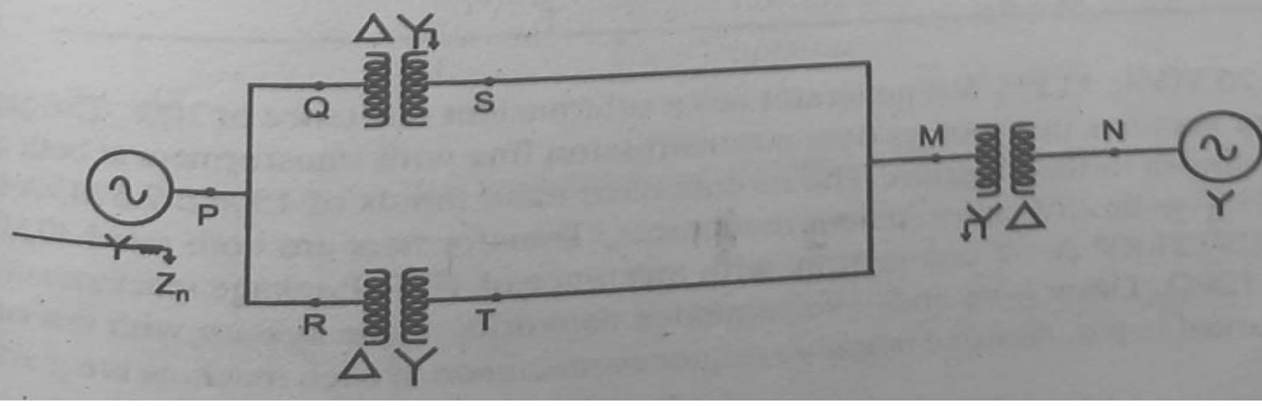
As we know that the neutral current is composed of zero sequence component current, for the zero sequence current to flow from the primary to secondary, definitely a path should exist from the primary neutral to the secondary neutral. Hence the zero sequence impedance offered by the transformer depends upon how the neutral of the primary and secondary winding are connected. The zero sequence networks of 3-$\phi$ transformers for various possible connections in primary and secondary are tabulated in the form of a table as shown.

From the figures, we can say that only when a definite neutral connection exists on both the primary and secondary windings, zero sequence impedance will come into picture. Otherwise the value of zero sequence impedance offered by the transformer is infinity.

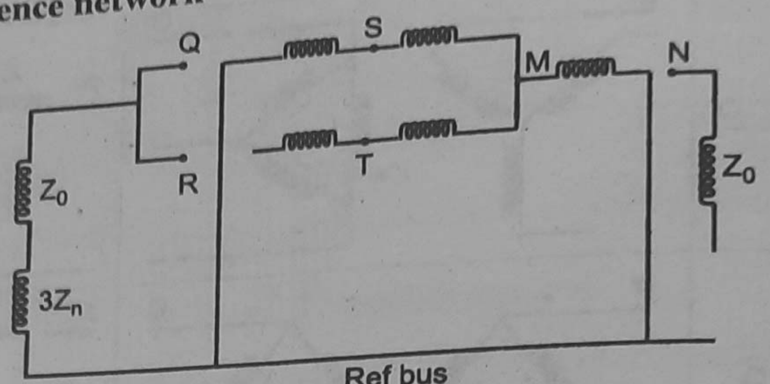
**Zero Sequence Equivalent Circuits of Three-Phase Transformers**



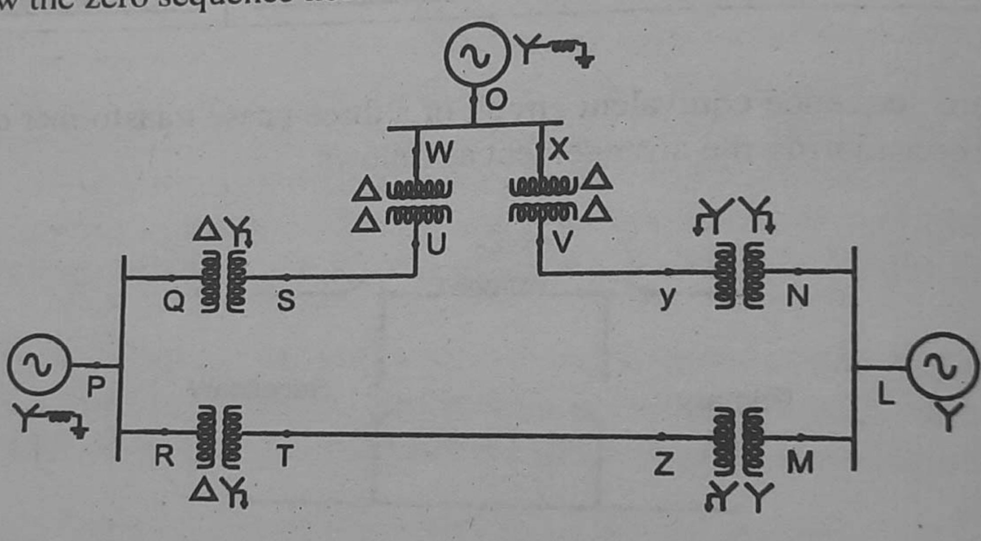
23. Draw the zero sequence network of the sample power system.



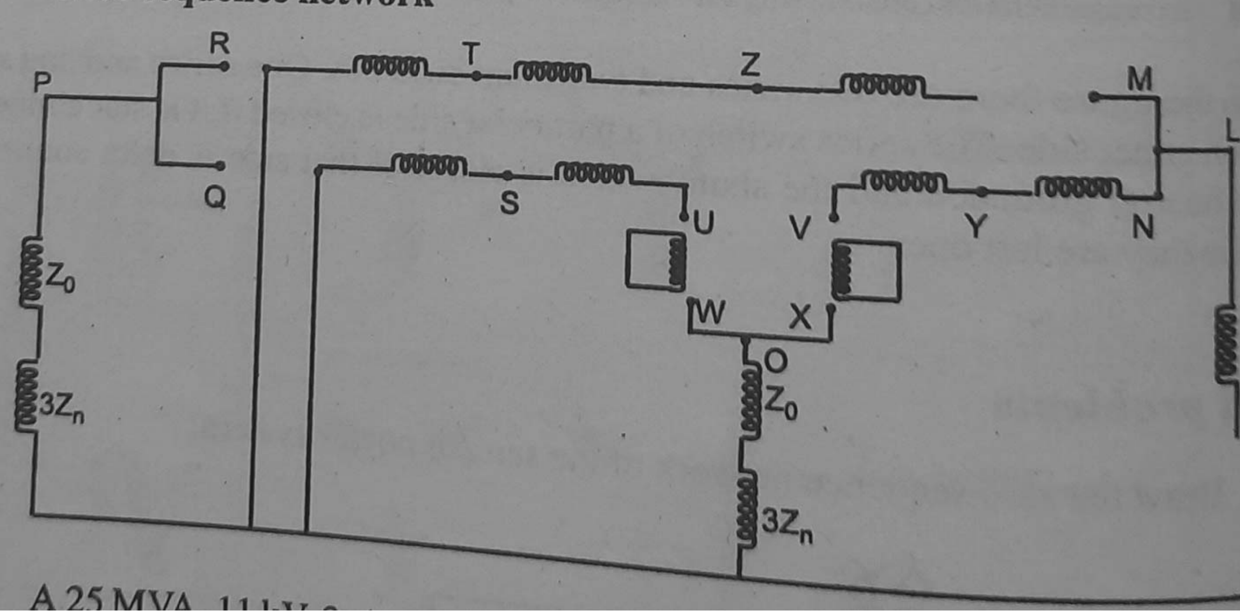

**Zero sequence network**




24. Draw the zero sequence network of the sample power system.

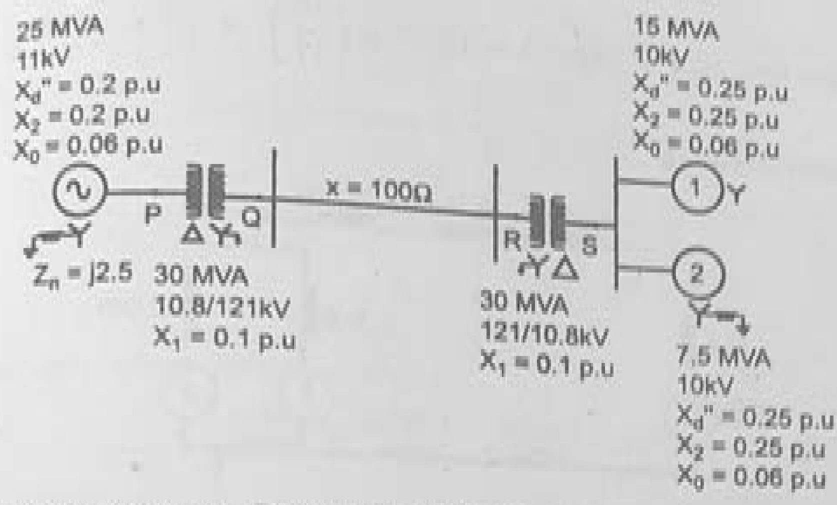



**Zero sequence network**




5. A 25 MVA 11kV

5. A 25 MVA, 11 kV, 3-$\phi$ generator has a subtransient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in the diagram. The motors have rated inputs of 15 and 7.5 MVA both 10 kV with 25% subtransient reactances. Transformers are both rated 30 MVA, 10.8/121 kV $\Delta$ - Y connection, with reactance of 10%. Leakage reactance of line is 100Ω. Draw +ve and −ve sequence networks of the system with reactances marked in p.u. Assume negative sequence reactance of each machine is equal to its




subtransient reactance. Draw the zero sequence network of the system assuming zero sequence reactances of generator and motor as 0.06 p.u. Current limiting reactors of 2.5Ω each are connected in the neutral of generator and motor no. 2. The zero sequence reactance of transmission is 100Ω.

**Solution:**

Let the generator ratings be chosen as the base values.

Base MVA 25

**Base kV**

Generator circuit - 11 kV

Transmission line - 123.24 kV

Motor circuit - 11 kV

**Positive and negative sequence networks**

Since the generator rating is chosen as the base values, $X_g = j0.2$.

**Transformer 1**

$$\text{p.u reactance} = 0.1 \times \left(\frac{10.8}{11}\right)^2 \times \frac{25}{30} = j0.0805$$

**Transmission line**

$$\text{Actual reactance} = j100 \text{ ohms}$$

$$\text{Base impedance} = \frac{(123.24)^2}{25} = 607 \text{ ohms}$$

$$\text{p.u. reactance} = \frac{\text{Actual reactance}}{\text{Base impedance}} = j0.1647$$

**Transformer 2**

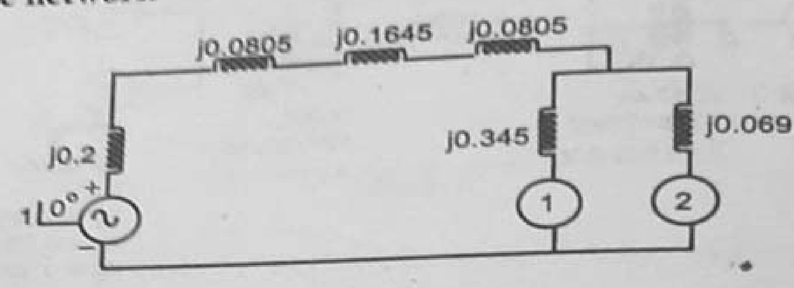
$$\text{p.u. reactance} = j0.0805 \text{ p.u.}$$

**Motors**

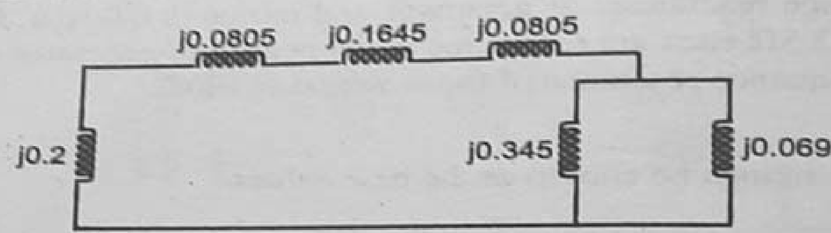
$$\text{p.u reactance of motor 1} = j0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = j0.345$$

$$\text{p.u reactance of motor 2} = j0.25 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = j0.69$$

**Positive sequence network**




**Negative sequence network**




**Zero sequence network calculations**
**Generator**

$$\text{p.u reactance} = 0.06 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{25}{25}\right) = 0.06\,\text{p.u.}$$

**Motors**

$$\text{p.u reactance of motot 1} = 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{15} = j0.083$$

$$\text{p.u reactance of motot 2} = 0.06 \times \left(\frac{10}{11}\right)^2 \times \frac{25}{7.5} = j0.1652$$

**Neutral Reactance**
**Generator**

$$\text{Base impedance} = \frac{11^2}{25} = 4.84\ \text{ohms}$$

$$Z_{n\,p.u} = \frac{j2.5}{4.84} = j0.5165$$

$$3Z_n = j1.5495\,\text{p.u.}$$

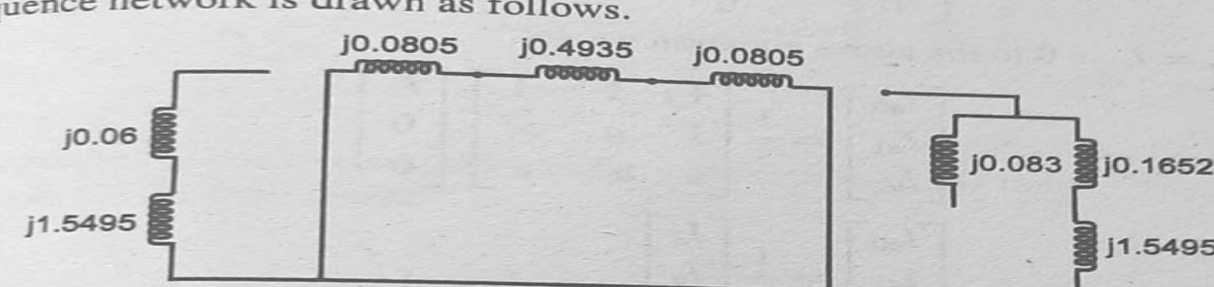
*Motor*

$$\text{Base impedance} = \frac{11^2}{25} = 4.84\ \text{ohms}$$

$$\text{p.u. reactance} = \frac{j2.5}{4.84} = j.5165$$

$$3Z_n = j1.5495\,\text{p.u.}$$

Zero sequence network is drawn as follows.



# UNIT-II Graph Theory

## 3.2 ORIENTED GRAPHS

In the electric transmission network, we are concerned with the interconnection of transmission lines, transformers and shunt reactors/capacitors that can be modeled in terms of two terminal passive components called *elements* as discussed in Chapter 2. The points of interconnection are called *buses*. The graph of a network represents the manner in which the passive elements and the buses are interconnected. Each of the two terminal elements is represented by a line segment called the *edge*. The edges will represent the interconnections or *topology* of the network. In the resulting graph, we will call the buses as *nodes*.

Figure 3.1(a) shows a network consisting of nine elements. Its graph is shown in Fig. 3.1(b) where the five nodes are numbered in parentheses. A direction may be associated with each edge of the graph in which case it is called an oriented or directed graph [Fig. 3.1(c)]. The directions are assigned consistent with the concept of associated reference directions for a two terminal passive element in circuit theory.

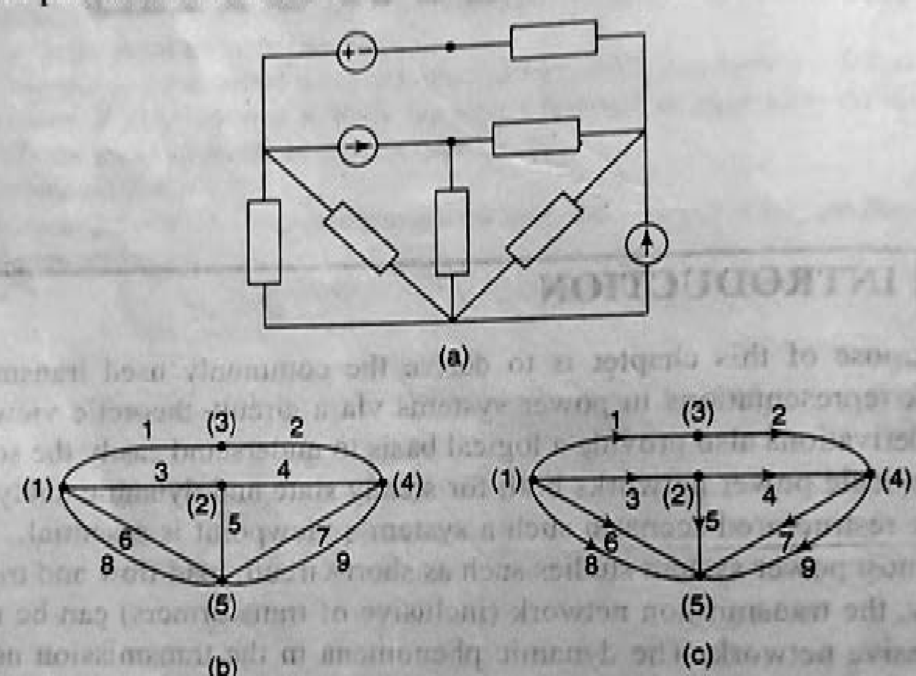



**Fig. 3.1** *A network and its oriented graph.*

### 3.2.1 Associated Reference Direction

Consider the element in Fig. 3.2(a) which may be a passive element, current or a voltage source. The associated reference directions are such that a positive current enters the + terminal of the voltage reference and leaves at the − terminal of the voltage reference. The oriented graph is shown in Fig. 3.2(b). If the element is purely passive and $v$ and $i$ are the phasors, then $v = zi$ where $z$ is the complex impedance of the element. The reference direction in the oriented graph is chosen to agree with the current direction [Fig. 3.2(b)]. If the element is a current source, then the positive orientation of the current source is chosen to agree with the reference direction in the graph (Fig. 3.3). Note that Terminal (1) has the + sign and Terminal (2) the − sign for the voltage across current source.


**Fig. 3.2** *Associated reference direction.*

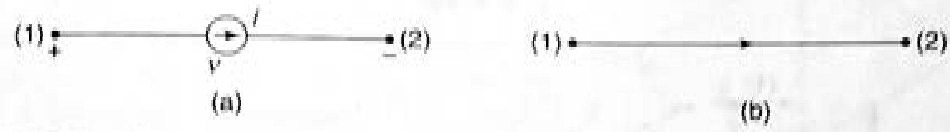



**Fig. 3.3** *Current source and its oriented graph.*

If the element is a voltage source, the orientation in the graph is chosen so that the arrow in the graph goes from the positive to the negative terminal of the voltage reference. The current, unlike in conventional circuit analysis, goes from + to − terminal (Fig. 3.4). Thus, while for the passive element the orientation of the graph is consistent with associated reference directions of circuit theory, for the current and voltage source it is not. If the element is purely passive, then Figs. 3.5 (a) and (b) describe the convention with $v = zi$ or $i = yv$.

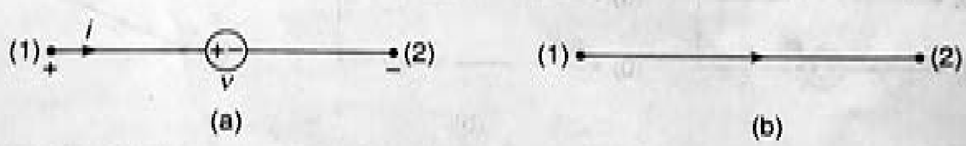



**Fig. 3.4** *Voltage source and its oriented graph.*

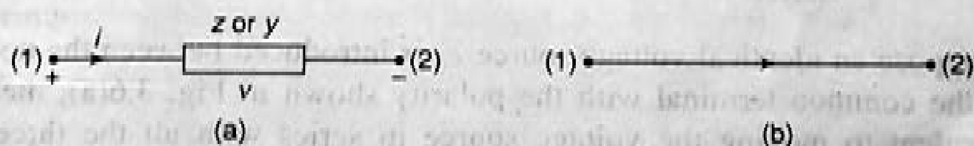



**Fig. 3.5** *Generalized circuit element and its oriented graph.*

## 3.3 PRIMITIVE IMPEDANCE AND ADMITTANCE MATRICES

'onsider a network of interconnected components. The passive components ıay be mutually coupled. The primitive impedance and admittance represen- ıtions are $v = z\,i$ where $v$ and $i$ are vectors, $z$ is the impedance matrix with $y$ ; the inverse of $z$. The diagonal elements of $z$ are self-impedances and the ff-diagonal elements are mutual impedances. If the $i$ and $j$ elements are utually coupled, then the corresponding $(i–j)$ and $(j–i)$ elements are nonzero.

**Example 3.1** Consider a four terminal network (e.g. three phases of a nerator which are mutually coupled) shown in Fig. 3.6 with all unequal utual impedances. For the passive network, the terminal relations are:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{3.1}$$

$$\boldsymbol{v} = \boldsymbol{z}\,\boldsymbol{i} \tag{3.2}$$

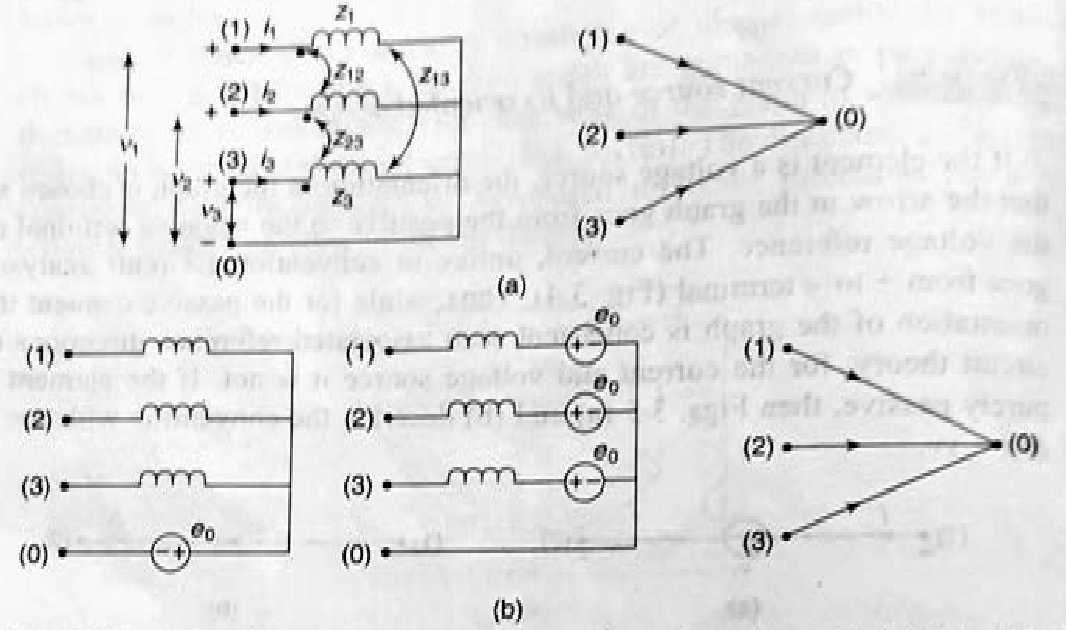



**Fig. 3.6** *(a) A three-phase network and its oriented graph. (b) Modified network*

Suppose an identical voltage source $e_0$ is introduced between the node (0) and the common terminal with the polarity shown in Fig. 3.6(a), then it is equivalent to moving the voltage source in series with all the three coils [Fig. 3.6(b)]. The graph will remain the same with each edge of the graph representing the Thevenin source. The terminal relations are now

$$\boldsymbol{v} = \boldsymbol{z}\,\boldsymbol{i} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e_0 \tag{3.3}$$

The admittance formulations for Eqs (3.2) and (3.3) are, respectively,

$$\boldsymbol{i} = \boldsymbol{y}\,\boldsymbol{v} \tag{3.4}$$

$$\boldsymbol{i} = \boldsymbol{y}\,\boldsymbol{v} - \boldsymbol{y} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e_0 \tag{3.5}$$

## 3.4 SYSTEM GRAPH FOR TRANSMISSION NETWORK

A power system is generally analyzed on a per-phase basis with balanced three-phase loads. Hence, only the positive sequence network is considered. The impedances in the per-phase equivalent are known as the positive sequence impedances. The calculation of these positive sequence impedances for a transmission line (both series impedance and shunt admittance) can be found in the standard texts as a first course in power system analysis. Topologically the positive sequence network is the same as the original single-line diagram of the network. Consider the graph of a certain passive network shown in Fig. 3.7.

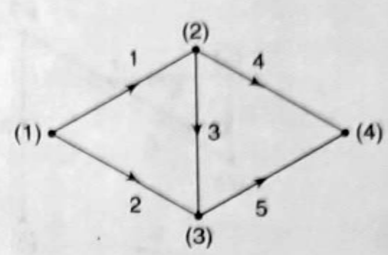



**Fig. 3.7** *Graph of a network.*

The primitive impedance $v$–$i$ relationship is given by

$$
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} z_{11} & 0 & 0 & 0 & 0 \\ 0 & z_{22} & 0 & 0 & 0 \\ 0 & 0 & z_{33} & 0 & 0 \\ 0 & 0 & 0 & z_{44} & 0 \\ 0 & 0 & 0 & 0 & z_{55} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} \tag{3.6}
$$

The primitive admittance $i$–$v$ relationship is $\boldsymbol{i} = \boldsymbol{y}\boldsymbol{v}$ where $\boldsymbol{y} = \boldsymbol{z}^{-1}$

$$
\boldsymbol{y} = \begin{bmatrix} z_{11}^{-1} & 0 & 0 & 0 & 0 \\ 0 & z_{22}^{-1} & 0 & 0 & 0 \\ 0 & 0 & z_{33}^{-1} & 0 & 0 \\ 0 & 0 & 0 & z_{44}^{-1} & 0 \\ 0 & 0 & 0 & 0 & z_{55}^{-1} \end{bmatrix} \tag{3.7}
$$

## 3.5 RELEVANT CONCEPTS IN GRAPH THEORY

Graph theory is a vast mathematical discipline with applications in various engineering fields. We need only a few basic concepts for our work in power systems.

A graph consisting of finite edges and nodes is called a *finite* graph. It is said to be *connected* if there exists a path between any two nodes of the graph. A subset of edges of the graph is called a *subgraph*. Certain degenerate



subsets such as an edge by itself or a node by itself are also consider subgraphs. A few subgraphs for the graph in Fig. 3.7 are shown in Fig. 3

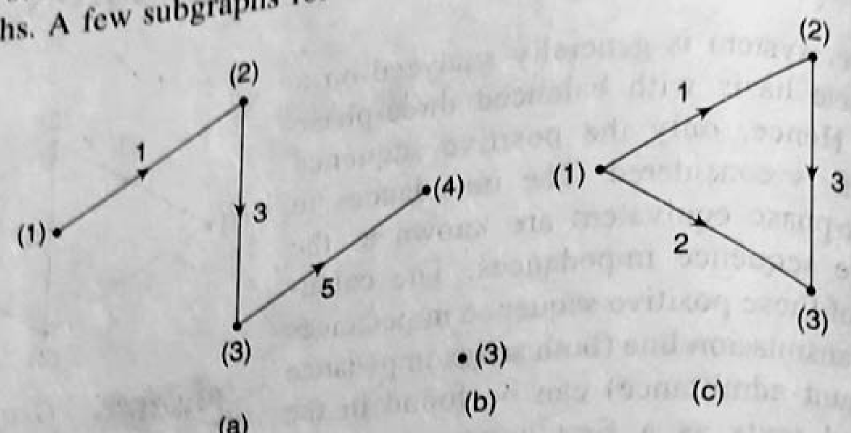



**Fig. 3.8** *Some subgraphs of the graph in Fig. 3.7.*

The number of edges incident at a node gives the degree of the node. Fig. 3.7 the degree of node (2) is 3. A subgraph with two endpoints (whi are the nodes) and all other nodes of degree two in the subgraph is called path. A path can traverse an edge at most once. For example, for Fig. 3.7 t subgraphs shown in Fig. 3.9 form the paths between nodes (1) and (4). T direction of the path that is arbitrarily drawn for each path is independent the orientation of its edges. In some paths, it may coincide with the orientati of some edges and in some it may be opposite to some of the edges.

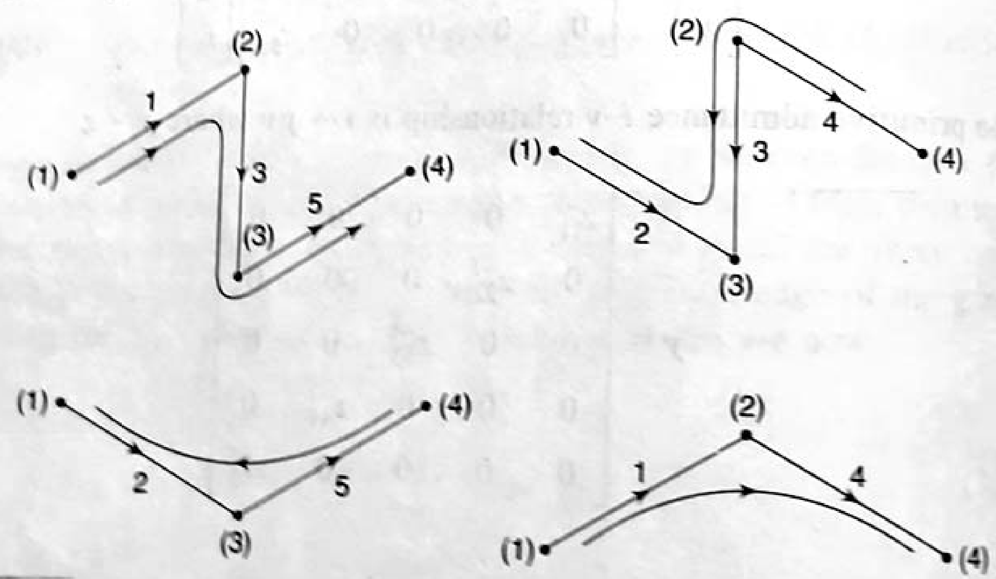



**Fig. 3.9** *Some paths of the graph in Fig. 3.7.*

### 3.5.1 Loop

A loop (circuit) is a connected subgraph with the degree of each of the nod in the sub-graph equal to two. The number of nodes and edges in a loop equal. A loop is also referred to as a closed path. For Fig. 3.1(c), some of t loops are (1, 2, 4, 3), (3, 5, 6), (6, 8), (1, 2, 7, 5, 3), etc. (shown in Fig. 3.1 A loop may also have an orientation that points away from one node a

finally goes back towards the same node along the elements of the loop. For the graph in Fig. 3.7 two of the loops are shown in Figs 3.11(a) and (b) along with their orientation. Fig. 3.11(c) is not a loop since the degree of node (2) in that subgraph is three.

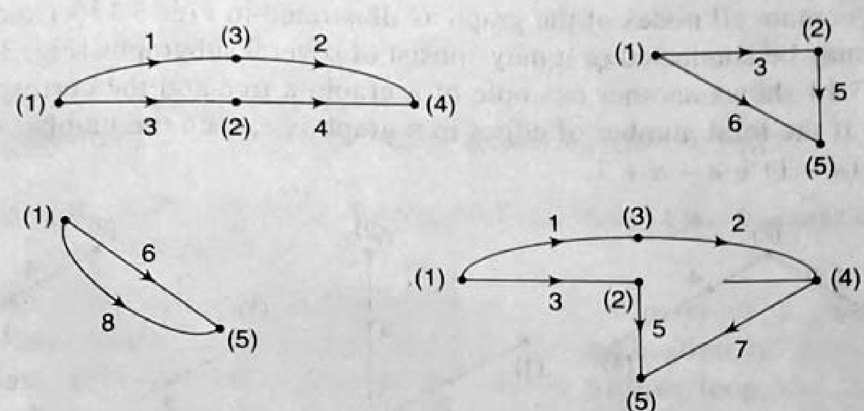



**Fig. 3.10** *Loops for the graph in Fig. 3.1(c).*

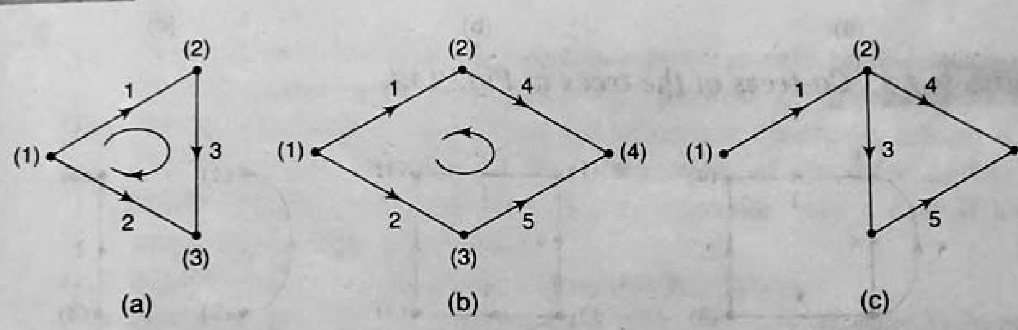



**Fig. 3.11** *Illustration of loops and subgraphs that are not loops.*

### 3.5.2 Tree and Co-tree

One of the important concepts in a linear graph is that of a *tree*. A *tree* is a subgraph that is connected, contains all nodes and has no loops. For example in Fig. 3.1(c), a tree can be formed by the elements (2, 5, 6, 7) or (2, 3, 4, 9). A few trees for Fig. 3.7 are shown in Fig. 3.12. In a tree, there is exactly one path between any two nodes. If the number of nodes in a graph is $n$, there are exactly $(n - 1)$ edges in a tree. The proof of this observation is obvious. The elements of the tree are called *tree branches*.

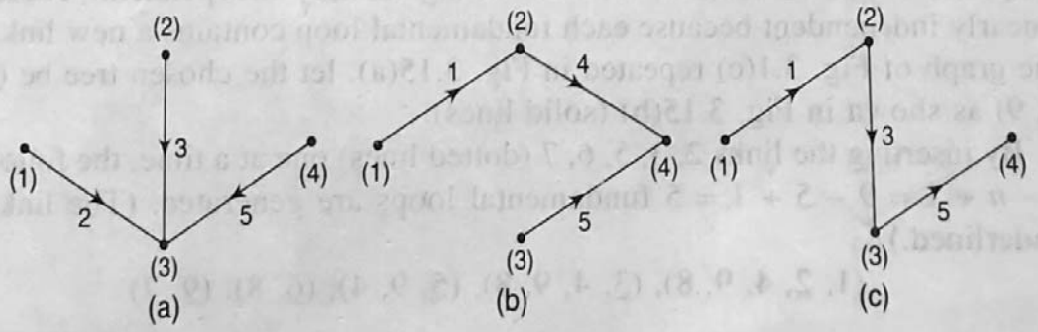



**Fig. 3.12** *Trees for the graph in Fig. 3.7.*

Those edges of the graph that are not in a tree form a *co-tree* and the edges of the co-tree are called *links* or *chords*. We use the term links. For each chosen tree, there is a *co-tree*. For the three trees chosen in Fig. 3.12 the corresponding co-trees are shown in Fig. 3.13. A co-tree does not in general contain all nodes of the graph as illustrated in Figs 3.13(a) and (b). A co-tree may be connected or it may consist of several subgraphs [Fig. 3.13(c)]. Figure 3.14 shows another example of a graph, a tree and the corresponding co-tree. If the total number of edges in a graph is $e$, then the number of links $\ell = e - (n - 1) = e - n + 1$.

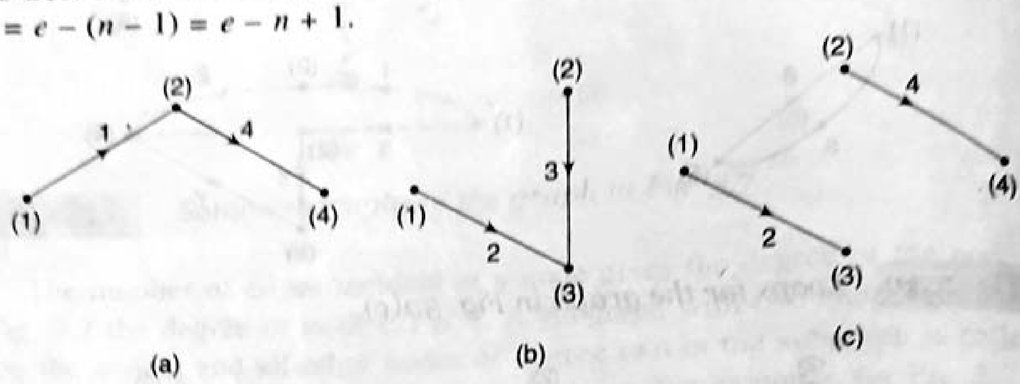



**Fig. 3.13** *Co-trees of the trees in Fig. 3.12.*

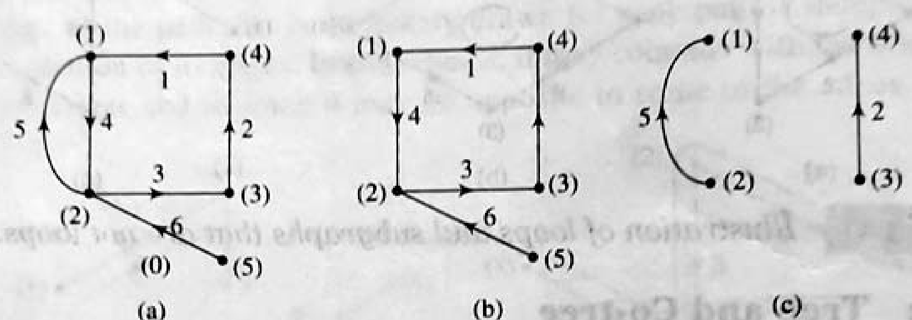



**Fig. 3.14** *(a) a graph (b) a tree (c) the corresponding co-tree*

### 3.5.3 Fundamental Loop

A *fundamental loop* for a graph is formed from the tree of the graph by inserting an appropriate link. For each link inserted, we create a new fundamental loop in the tree. There will be in all $(e - n + 1)$ fundamental loops for a chosen tree, all of these being linearly independent. These are linearly independent because each fundamental loop contains a new link. For the graph of Fig. 3.1(c) repeated in Fig. 3.15(a), let the chosen tree be (1, 4, 8, 9) as shown in Fig. 3.15(b) (solid lines).

By inserting the links 2, 3, 5, 6, 7 (dotted lines) one at a time, the following $e - n + 1 = 9 - 5 + 1 = 5$ fundamental loops are generated. (The links are underlined.)

$$(1, \underline{2}, 4, 9, 8), (\underline{3}, 4, 9, 8), (\underline{5}, 9, 4), (\underline{6}, 8), (\underline{9}, 7)$$

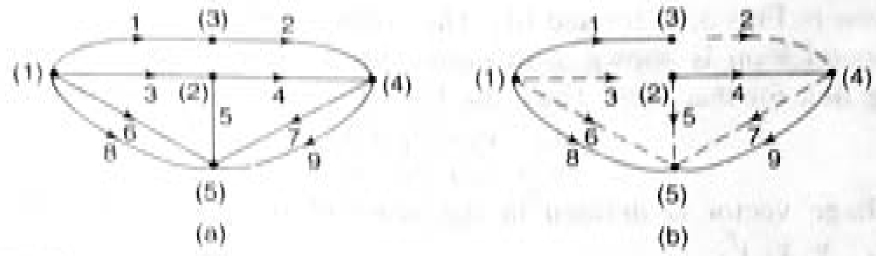


**Fig. 3.15** *Graph, tree (solid) and the links (dotted).*

### 3.5.4 Kirchhoff's Voltage Law and the Fundamental Loop Matrix

We now state an important topological property of a graph, namely the *Fundamental Loop Matrix* through the application of Kirchhoff's voltage law (KVL). It states that for any closed path or loop, the algebraic sum of voltages around the loop is zero. We write KVL systematically for the fundamental loops as follows:

(i) Select a tree.

(ii) For each fundamental loop assign a positive reference direction to agree with the orientation associated with the link for that loop.

(iii) Going around the loop along the reference direction, assign a + sign to the voltage of the edge if the orientation of the edge agrees with the reference direction, a – sign if it is opposite, and a zero if the edge is not contained in that loop.

(iv) Repeat Step (iii) for all the fundamental loops.

(v) Arrange the voltage vector such that the tree-branch voltages appear first and the link voltages afterwards.

(vi) The resulting matrix of +1, –1 and 0 entries is called the *Fundamental Loop Matrix.*

**Example 3.2** Consider the graph of Fig. 3.16(a) and the tree (1, 3, 4) in Fig. 3.16(b). The fundamental loops obtained by inserting links 2 and 5

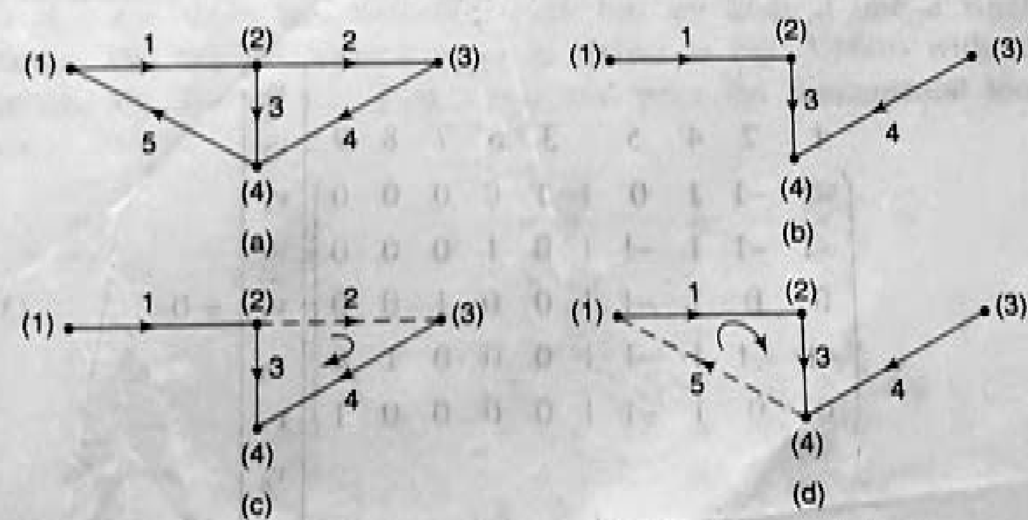


**Fig. 3.16** *(a) graph, (b) tree, (c) and (d) fundamental loops.*

are shown in Figs 3.16 (c) and (d). The positive reference direction for each fundamental loop is shown with dotted lines to coincide with that of the defining link for that loop. Thus, the KVL for the two loops are written as

$$v_2 - v_3 + v_4 = 0 \tag{3.8a}$$

$$v_1 + v_3 + v_5 = 0 \tag{3.8b}$$

The voltage vector is defined in the order of tree branches and links as $[v_1 \; v_3 \; v_4 \; \vdots \; v_2 \; v_5]^T$.

The KVL equations can now be put in a matrix form as

$$\begin{array}{c} \\ \\ 2 \\ 5 \end{array}\begin{array}{c} \text{Tree branches} \quad \text{Links} \\ \begin{array}{ccc|cc} 1 & 3 & 4 & 2 & 5 \end{array} \\ \left[\begin{array}{ccc|cc} 0 & 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \begin{bmatrix} v_1 \\ v_3 \\ v_4 \\ \hline v_2 \\ v_5 \end{bmatrix} = 0 \tag{3.9}$$

**Example 3.3** From Fig. 3.1(c), choose the tree (1, 2, 4, 5) and write the KVL in matrix form.

*Solution*

The tree is shown in Fig. 3.17 in solid lines and the links in dotted lines.

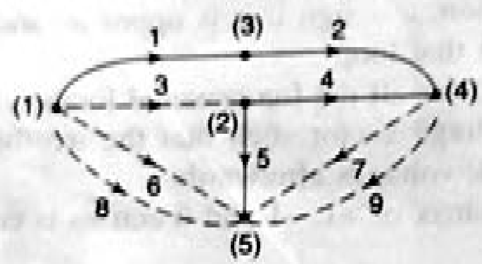



**Fig. 3.17** *Network with tree branches and links.*

The KVL equations can be written by inspection as

$$\begin{array}{c} \begin{array}{cccc|ccccc} 1 & 2 & 4 & 5 & 3 & 6 & 7 & 8 & 9 \end{array} \\ \left(\begin{array}{cccc|ccccc} -1 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right) \end{array} \begin{bmatrix} v_1 \\ v_2 \\ v_4 \\ v_5 \\ \hline v_3 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \end{bmatrix} = 0 \tag{3.10}$$

*Generalization*

If the preceding procedure is followed for a general finite graph, then the KVL equations can be written in a form

$$e - n + 1 \begin{bmatrix} \overset{n-1}{C_b} : \overset{e-n+1}{U} \end{bmatrix} \begin{bmatrix} v_b \\ \hline v_\ell \end{bmatrix} = 0 \tag{3.11}$$

that is

$$Cv = 0$$

where

$C_b$ is a $(e - n + 1) \times (n - 1)$ matrix.

$U$ is a $(n - 1)$ square matrix.

$v_b$ is sub-vector of order $(n - 1)$ corresponding to the tree-branch variables.

$v_\ell$ is a sub-vector of order $e - n + 1$ corresponding to the link variables.

$C$ is called the *fundamental loop matrix*.

The existence of the unity sub-matrix in $C$ is easily verified from the fact that,

(i) each fundamental loop contains one link only, and

(ii) the positive orientation of the loop coincides with the orientation of the link for that particular loop.

In general, the entries $C$ are such that,

(i) $c_{ij} = +1$ if the element corresponding to the $j$th column is in the fundamental loop defined by the link in the $i$th row and their orientations agree.

(ii) $c_{ij} = -1$ if the element corresponding to the $j$th column is in the fundamental loop defined by the link in the $i$th row but their orientations are opposite.

(iii) $c_{ij} = 0$ if the element corresponding to the $j$th column is **not** in the fundamental loop defined by the link in the $i$th row.

**Example 3.4** For the transmission network shown in Fig. 3.18(a), assume that the shunt admittances at each bus are lumped into a single admittance. The oriented system graph is shown in Fig. 3.18(b) with (0) representing the ground bus. Pick a tree and write the fundamental loop matrix $C$.

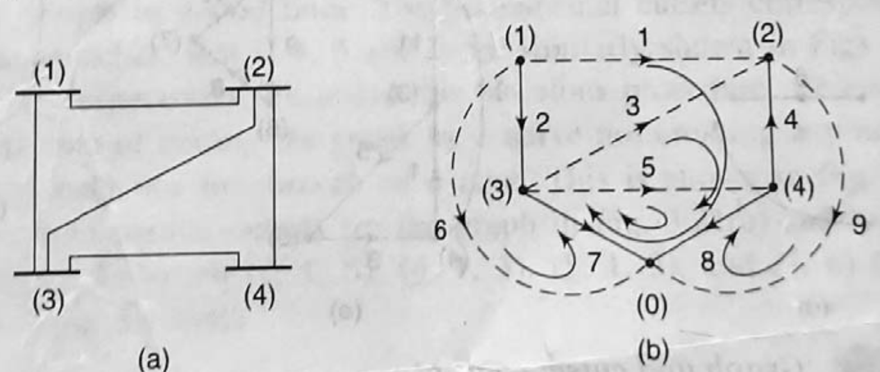



**Fig. 3.18** *Transmission, network, graph tree and co-tree*

*Solution*

The following tree is chosen with the tree branches (2, 4, 7, 8), shown by solid lines. The links are (1, 3, 5, 6, 9) shown by dotted lines. The orientations for the fundamental loops are shown with dotted lines. The $C$ matrix is written as

$$
\begin{array}{c} \\ \\ C = \end{array}
\begin{array}{c} \\ \\ 1 \\ 3 \\ 5 \\ 6 \\ 9 \end{array}
\begin{array}{c}
\text{Tree branches} \qquad\qquad \text{Links} \\
\begin{array}{cccc|ccccc} 2 & 4 & 7 & 8 & 1 & 3 & 5 & 6 & 9 \end{array} \\
\left(\begin{array}{cccc|ccccc}
-1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\
-1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1
\end{array}\right)
\end{array}
\qquad (3.12)
$$

### 3.5.5 Fundamental Cutset

Another basic concept in graph theory is that of a cutset. A cutset of a connected graph is defined as the minimal set of elements whose removal leaves the graph in exactly two parts. Consider the graph in Fig. 3.19(a). Removal of elements (3, 4, 5, 6, 7) [Fig. 3.19(b)] leaves the graph in three

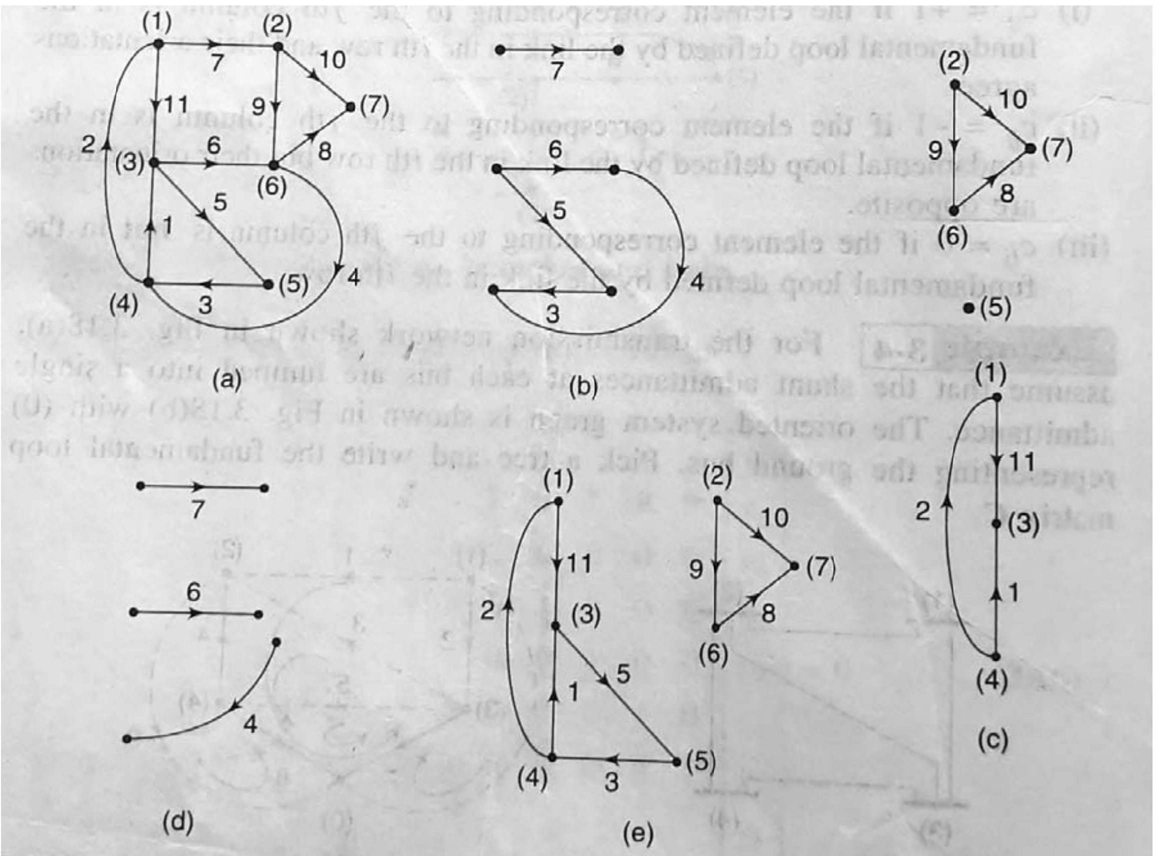



**Fig. 3.19** *Graph and cutset concept*

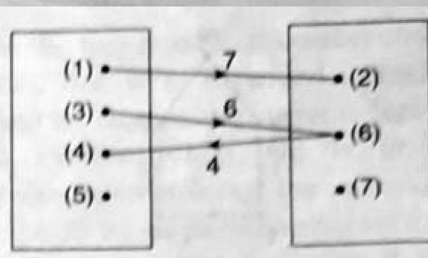



**Fig. 3.20** *Further illustration of cutset of Fig. 3.19(d).*

parts as shown in Fig. 3.19(c). Note that node (5) by itself constitutes a subgraph. Hence (3, 4, 5, 6, 7) does not form a cutset. On the other hand, removal of (4, 6, 7) [Fig. 3.19(d)] leaves it in two parts as shown in Fig. 3.19(e). Hence (4, 6, 7) is a cutset. The elements of the cutset can also be selected by "cutting" the graph with a curved (dotted) line not passing through any node and dividing the graph in two connected subgraphs. The cutset (4, 6, 7) also divides the nodes of the graph into two groups, one group consisting of nodes (1), (3), (4), (5) and the other group consisting of nodes (2), (6) and (7). The edges of the cutset connect the nodes between the two groups as shown in Fig. 3.20. The reader may verify the other cutsets in Fig. 3.19(a) as (2, 11, 7), (1, 2, 3, 4), (4, 6, 9, 10), (2, 4, 6, 11), etc. Just as the concept of fundamental loops is associated with a link, so is the concept of fundamental cutsets associated with a tree branch that we discuss next.

The tree is a connected subgraph of a given graph. Removal of any tree branch leaves the tree in two parts, each part having a certain number of nodes. We thus have two groups of nodes. The edges of the graph connecting these two groups of nodes are called *fundamental cutsets* and correspond to that *particular tree branch*. The edges of the cutset are the particular tree branch and other links that connect the two groups of nodes. Thus, for each treebranch we have an associated fundamental cutset. Altogether, we have $(n - 1)$ fundamental cutsets in all since a tree in an $n$ node graph has $(n - 1)$ edges.

Consider the graph in Fig. 3.21(a). Let the tree branches be (2, 4, 5, 7) which constitutes a connected graph [Fig. 3.21 (b)]. Removal of tree-branch 2 in the tree divides the nodes into two groups of nodes as shown in Fig. 3.21(c). We then insert all the possible links of the graph between the two nodes. This constitutes a fundamental cutset associated with branch 2. For convenience, the tree-branch 2 is shown in a solid line and the other links are shown in dotted lines. The fundamental cutsets corresponding to other tree-branches, that is 4, 5 and 7 are similarly shown in Figs 3.21(d), (e), and (f), respectively. To avoid this laborious procedure, we can follow the simple rule of cutting the graph by a curve not crossing any node such that it cuts only one tree-branch *at a time*. This is shown in Fig. 3.21(g). Thus, the fundamental cutsets for the graph in Fig. 3.21(a) and the chosen tree in Fig. 3.21(b) are (2, 1, 6), (4, 1, 3), (5, 1, 3), and (7, 6) (the tree-branches are underlined).

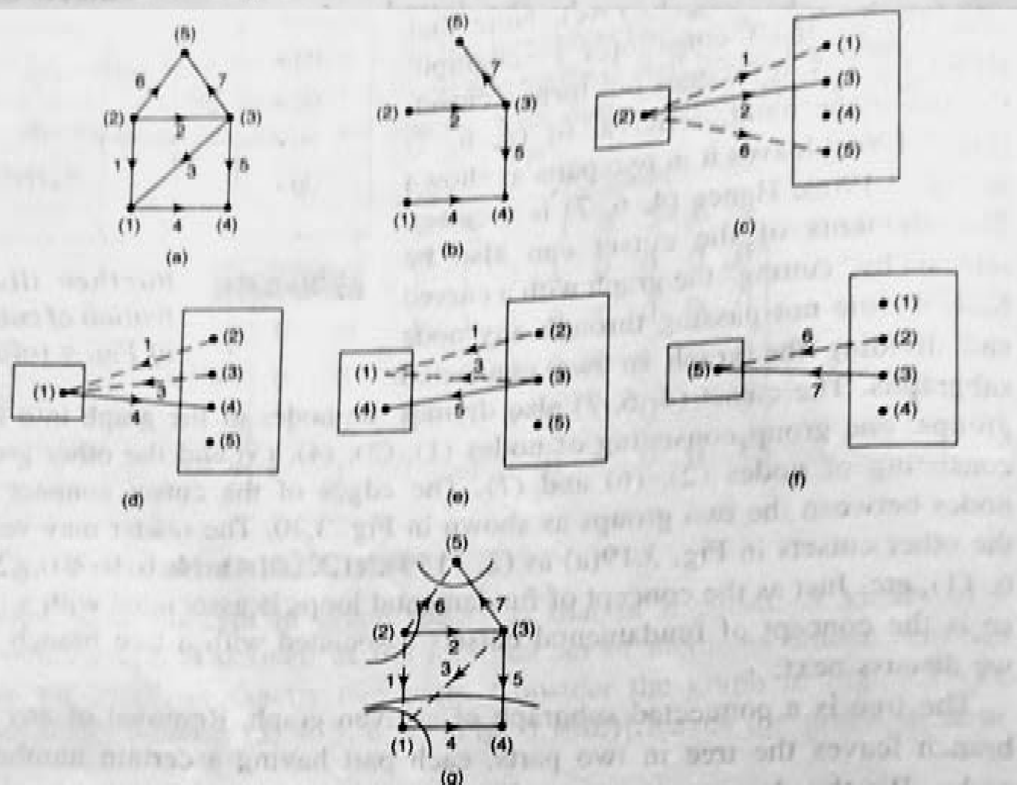



**Fig. 3.21** *Network and fundamental cutsets.*

It is of interest to remark here that a set of linearly independent cutsets can also exist which cannot be determined by a tree. As an example, consider the graph in Fig. 3.22. The elements incident on each node is a cutset and the edges of each cutset are the ones cut by a curved line. But as we shall see later, only $(n - 1)$ cutsets in an $n$ node graph constitute a linearly independent set of cutsets.

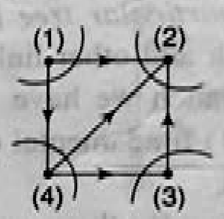



**Fig. 3.22** *Cutsets not generated by a tree.*

## 3.5.6 Kirchhoff's Current Law (KCL) and the Fundamental Cutset Matrix

Just as in the case of fundamental loops we shall use KCL to derive another important topological relationship. We shall use the same graph (Fig. 3.16) as for KVL and is reproduced in Fig. 3.23. Choose (1, 3, 4) as the tree.

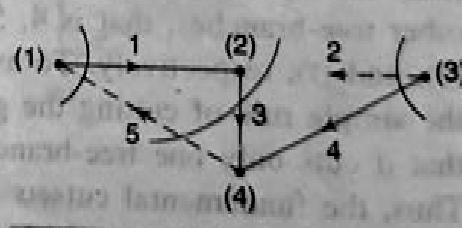



**Fig. 3.23** *Fundamental cutsets and KCL*

The three fundamental cutsets associated with tree-branches are shown by the dotted curved lines. These are (1, 5), (2, 3, 5) and

(2, $\underline{4}$). The underlined element corresponds to the tree branch. If corresponding to each fundamental cutset, the curved dotted line were extended to form a closed surface, then KCL states that the algebraic sum of the currents leaving a closed surface is zero. To apply KCL systematically, we define the orientation of each cutset to coincide with the orientation of the associated tree branch. In writing KCL we give a + sign to an edge of the cutset if its orientation agrees with the orientation of the cutset and a − sign if it is opposite. Application of KCL to each of the three cutsets in Fig. 3.23 gives

$$i_1 - i_5 = 0 \tag{3.13a}$$

$$-i_2 + i_3 - i_5 = 0 \tag{3.13b}$$

$$i_2 + i_4 = 0 \tag{3.13c}$$

Arranging Eqs. (3.13a), (3.13b) and (3.13c) in matrix form we get

$$\begin{array}{cc} & \begin{array}{ccccc} \multicolumn{3}{c}{\text{Tree branches}} & \multicolumn{2}{c}{\text{Links}} \\ 1 & 3 & 4 & 2 & 5 \end{array} \\ \text{Tree branches} \begin{array}{c} 1 \\ 3 \\ 4 \end{array} & \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \end{array} \left[\begin{array}{c} i_1 \\ i_3 \\ i_4 \\ \hline i_2 \\ i_5 \end{array}\right] = 0 \tag{3.14}$$

As in the case of the fundamental loop matrix, the current variables associated with the tree branches are listed first followed by the variables associated with the links.

**Example 3.5** For the graph in Fig. 3.24 and tree (1, 2, 4, 5), write the KCL.

*Solution*

The fundamental cutsets are shown in Fig. 3.24 along with their positive orientations shown by an arrow in the direction coinciding with that of the tree branch. The KCL is written as

$$\begin{array}{c} \begin{array}{ccccccccc} \multicolumn{4}{c}{\text{Tree branches}} & \multicolumn{5}{c}{\text{Links}} \\ 1 & 2 & 4 & 5 & 3 & 6 & 7 & 8 & 9 \end{array} \\ \left[\begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}\right] \end{array} \left[\begin{array}{c} i_1 \\ i_2 \\ i_4 \\ i_5 \\ \hline i_3 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \end{array}\right] = 0 \tag{3.15}$$

***Generalization***

**For a general graph we can write the KCL as**

$$n-1 \left[\begin{array}{c|c} \overset{n-1}{U} & \overset{e-n+1}{B_l} \end{array}\right] \left[\begin{array}{c} i_b \\ \hline i_l \end{array}\right] = 0 \tag{3.16}$$

**Since each fundamental cutset contains only one tree branch, the nature of the unity matrix $U$ is self-evident. In a more compact form**

$$\boldsymbol{B\,i = 0} \tag{3.17}$$

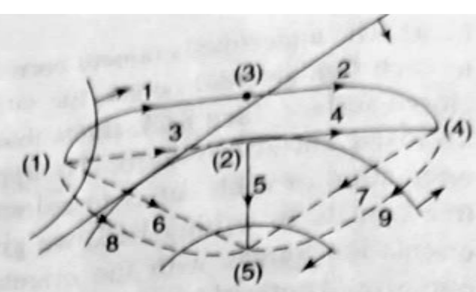



**Fig. 3.24** *Fundamental cutsets*

where

$$B = [\ U \mid B_f\ ] \tag{3.18}$$

and $i$ is the vector of currents arranged in the order of tree branch and link currents. $B$ is called the *fundamental cutset matrix*. It has unity submatrix of order $(n-1)$ in the leading position and the matrix $B_\ell$ of order $(n-1) \times (e-n+1)$ in the trailing position. Each row is identified with a tree branch. The entries of the matrix $B$ are such that

$b_{ij} = 1$, if the orientation of the element corresponding to the $j$th column agrees with the orientation of the tree branch corresponding to the $i$th row.

$b_{ij} = -1$, if the orientation of the element in the $j$th column is opposite to the orientation of the tree branch corresponding to the $i$th row.

$b_{ij} = 0$, if the orientation corresponding to the $j$th column does not belong to the tree branch corresponding to the $i$th row.

**Example 3.6** For the graph in Fig. 3.18(b) and the chosen tree (2, 4, 7, 8), write the $B$ matrix.

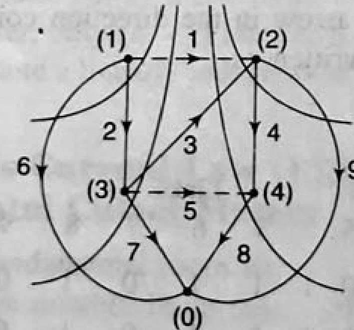



**Fig. 3.25** *Graph and fundamental cutsets for the transmission network of Fig. 3.18.*

*Solution*

The graph is redrawn in Fig. 3.25 with the curved lines defining the fundamental cutsets. The $B$ matrix is written by inspection as

$$B = \begin{array}{c} \\ \\ 2 \\ 4 \\ 7 \\ 8 \end{array} \begin{array}{c} \text{Tree branches} \qquad\qquad \text{Links} \\ \begin{array}{cccc|ccccc} 2 & 4 & 7 & 8 & 1 & 3 & 5 & 6 & 9 \end{array} \\ \left(\begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 & 0 & 1 \end{array}\right) \end{array} \tag{3.19}$$

### 3.5.7 Incidence or Vertex Matrix

One of the characterizations of a graph is the *incidence* matrix. The edges incident to a node in a graph is called the *incidence set*. Thus a connected graph has as many incidence sets as there are nodes. We can write KCL at each of these nodes giving a + sign to the currents leaving the node and a-sign to the currents entering the node. Alternatively, we can interpret each incidence set as a cutset with a line enclosing the node and the positive orientation of the cutset outwards from the dotted closed line (see Fig. 3.26). The KCL equations for nodes (1)–(4) can be written as

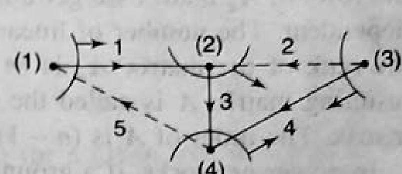



**Fig. 3.26** *Incidence sets in a graph.*

$$i_1 - i_5 = 0 \tag{3.20a}$$

$$-i_1 - i_2 + i_3 = 0 \tag{3.20b}$$

$$i_2 + i_4 = 0 \tag{3.20c}$$

$$-i_3 - i_4 + i_5 = 0 \tag{3.20d}$$

In matrix form Eqs (3.20a) to (3.20d) can be written as

$$i_1 - i_5 = 0 \tag{3.20a}$$
$$-i_1 - i_2 + i_3 = 0 \tag{3.20b}$$
$$i_2 + i_4 = 0 \tag{3.20c}$$
$$-i_3 - i_4 + i_5 = 0 \tag{3.20d}$$

In matrix form Eqs (3.20a) to (3.20d) can be written as

$$\begin{array}{cc} & \text{Edges} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Nodes} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 \end{pmatrix} \end{array} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = 0$$

$$A_a i = 0 \tag{3.21}$$

In general, the order of $A_a$ is $n \times e$ where $n$ = number of nodes and $e$ = number of edges in the graph. $A_a$ is called the *node to branch incidence matrix* or *augmented incidence matrix*. The entries of $A_a$ are such that



$(a_{ij})_a = +1$, if the edge corresponding to the $j$th column is incident to the node corresponding the $i$th row and is directed away from it.

$(a_{ij})_a = -1$ if the edge corresponding to the $j$th column is incident to the node corresponding to the $i$th row and is directed towards the node.

$(a_{ij})_a = 0$ if the edge corresponding to the $j$th column is not incident to the node corresponding to the $i$th row.

It may be observed that since each element is incident on two nodes, the columns of the $A_a$ matrix have each $a$ + 1 and $a$ – 1 entry. If we add up all the rows of $A_a$ matrix we get a zero row. This indicates the rows are linearly dependent. The number of linearly independent rows is $n - 1$ or we say that the rank of the matrix $A_a$ is $(n - 1)$. We can delete any one row and the resulting matrix $A$ is called the *reduced–incidence* or simply the *incidence* matrix. The order of $A$ is $(n - 1) \times e$.

In power networks, if a ground bus is present, it is generally the reference bus and the node corresponding to it is generally deleted in writing the $A$ matrix. If the network has no connection to ground, one of the nodes is taken as reference and then deleted in writing the $A$ matrix.

**Example 3.7** Write the reduced incidence matrix for the transmission network in Fig. 3.18. Choose the ground bus (0) as reference bus.

*Solution*

By inspection, the matrix $A$ is written as

$$A = \begin{array}{cc} & \text{Edges} \\ & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \text{Nodes} \begin{array}{c} (1) \\ (2) \\ (3) \\ (4) \end{array} & \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & +1 & -1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

### 3.5.8 Interrelationships between the Matrices A, B, C and the Network Graph

In $A$ matrix the columns corresponding to the edges were arranged sequentially. They can be written in any particular order. In fact, one of the ways is to arrange the columns in the order of tree branches and links for a given tree in the graph. Thus, we can write $A$ as

$$A = \overset{\text{Tree branches}}{[A_b} \mid \overset{\text{Links}}{A_\ell]} \tag{3.22}$$

The following properties are now stated without a rigorous proof and illustrated for some examples. The proofs can be found in texts on graph theory.

**Property 1**

For a given tree of a graph each row of the fundamental loop matrix $C$ is orthogonal to each row of the fundamental cutset matrix $B$. Mathematically this relationship implies

$$BC^T = C^T B = 0 \tag{3.23}$$

Since $B = [U \mid B_\ell]$ and $C = [C_b \mid U]$, it follows

$$[U \mid B_\ell]\left[\frac{C_b^T}{U}\right] = 0 \tag{3.24}$$

Therefore, $C_b^T = -B_\ell$ which is the same as

$$C_b = -B_\ell^T \tag{3.25}$$

This is a very important result. It tells us that for a given tree of a graph, if the fundamental loop matrix $C$ is known, the fundamental cutset matrix is also known and vice-versa. This relationship can be verified from Eq. (3.25).

**Property 2**

Let the incidence matrix $A$ be arranged in the order of tree branches and links for a given tree, i.e.

$$A = n-1\overset{\quad n-1 \qquad e-n+1}{[\ A_b \quad \mid \quad A_\ell\ ]} \tag{3.26}$$

It can be shown that $A_b$ is nonsingular. Furthermore, the fundamental cutset matrix for the given tree is given by

$$\begin{aligned} B &= A_b^{-1} A \\ &= A_b^{-1}[A_b \mid A_\ell] \\ &= [U \mid A_b^{-1}A_\ell] \end{aligned} \tag{3.27}$$

since

$$B = [U \mid B_\ell]$$

we have

$$B_\ell = A_b^{-1} A_\ell$$

This important result tells us that by choosing a tree and writing the incidence matrix by inspection (or computer generated) we can obtain the fundamental cutset matrix $B$ and also the fundamental loop matrix $C$ from Property 1.

**Example 3.8** Consider the graph shown in Fig. 3.27. Choose the tree whose branches are (1, 3, 5). Find the fundamental cutset and loop matrices $B$ and $C$ using the incidence matrix $A$.

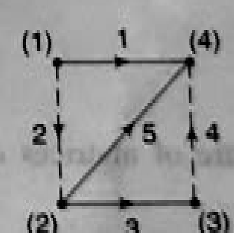



**Fig. 3.27** *Oriented graph*

*Solution*

Choosing (2) as the reference node, we write the reduced incidence matrix $A$ as

$$\begin{array}{c} \\ \\ \text{Nodes} \\ A = \begin{array}{c}(1)\\(3)\\(4)\end{array} \end{array} \begin{array}{c} \begin{array}{cc}\text{Tree branches} & \text{Links}\end{array} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & -1 \end{array}\right) \end{array} \tag{3.28}$$

$$= [A_b | A_\ell]$$

$$A_b^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \tag{3.29}$$

Therefore,

$$A_b^{-1} A_\ell = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{3.30}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$$

Hence,

$$\begin{array}{c} \\ \\ B = \begin{array}{c}1\\3\\5\end{array} \end{array} \begin{array}{c} \begin{array}{cc}\text{Tree branches} & \text{Links}\end{array} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right) \end{array} \tag{3.31}$$

$$= [U | B_\ell]$$

Since $C_b = -B_\ell^T$, we have

$$\begin{array}{c} \\ \\ C = \begin{array}{c}2\\4\end{array} \end{array} \begin{array}{c} \begin{array}{cc}\text{Tree branches} & \text{Links}\end{array} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left(\begin{array}{ccc|cc} -1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{array}\right) \end{array} \tag{3.32}$$

The nature of matrices of $B$ and $C$ can be independently verified from the graph.

## Bus Admittance matrix :-

- The Bus Admittance matrix is formed and used in load flow, short circuit and transient stability studies.

- It relates bus currents with bus voltages.

$$[I] = [Y][V]$$

where
$[I]$ : vector of bus currents, $(nb \times 1)$
$[V]$ = vector of bus voltages $(nb \times 1)$
$[Y]$ = bus admittance matrix $(nb \times nb)$
$nb$ : number of buses.

- It is a square matrix.

- It is a symmetric matrix. But in the networks having phase shifting transformer, it is non-symmetric.

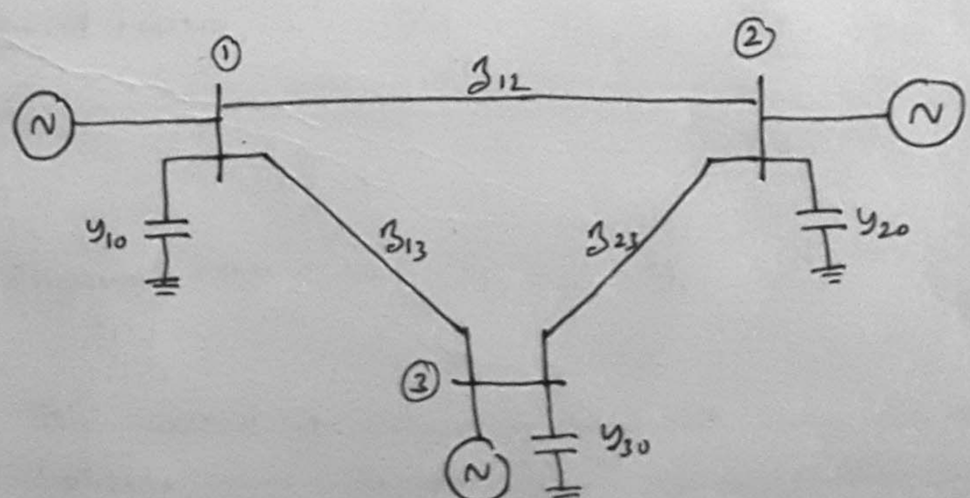
- It will be ~~...~~ singular, if there is no shunt connections such as line charging admittance, shunt capacitance etc to the ground.

- It will be non-singular, if there are shunt connections to the ground.

- It can be formed either by inspection or by analytical method.

## Formation of bus admittance by the method of inspection :-

Consider the transmission system shown in fig. The line impedances joining buses 1, 2 and 3 are denoted by $z_{12}$, $z_{23}$ and $z_{31}$ respectively. The corresponding line admittances are $y_{12}$, $y_{23}$ and $y_{31}$



The total capacitance susceptances at the buses are represented by $y_{10}$, $y_{20}$, and $y_{30}$. Applying KCL at each bus, we get

$$I_1 = V_1 \cdot y_{10} + (V_1 - V_2) \cdot y_{12} + (V_1 - V_3) \cdot y_{13}$$

$$I_2 = V_2 \cdot y_{20} + (V_2 - V_1) \cdot y_{21} + (V_2 - V_3) \cdot y_{23}$$

$$I_3 = V_3 \cdot y_{30} + (V_3 - V_1)\, y_{31} + (V_3 - V_2) \cdot y_{32}.$$

In matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{20}+y_{12}+y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{30}+y_{13}+y_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where

$Y_{11} = y_{10} + y_{12} + y_{13}$

$Y_{22} = y_{20} + y_{12} + y_{23}$

$Y_{33} = y_{30} + y_{13} + y_{23}$ are

the self admittances forming the diagonal terms and

$Y_{12} = Y_{21} = -y_{12}$ ; $Y_{13} = Y_{31} = -y_{13}$ ; $Y_{23} = Y_{32} = -y_{23}$

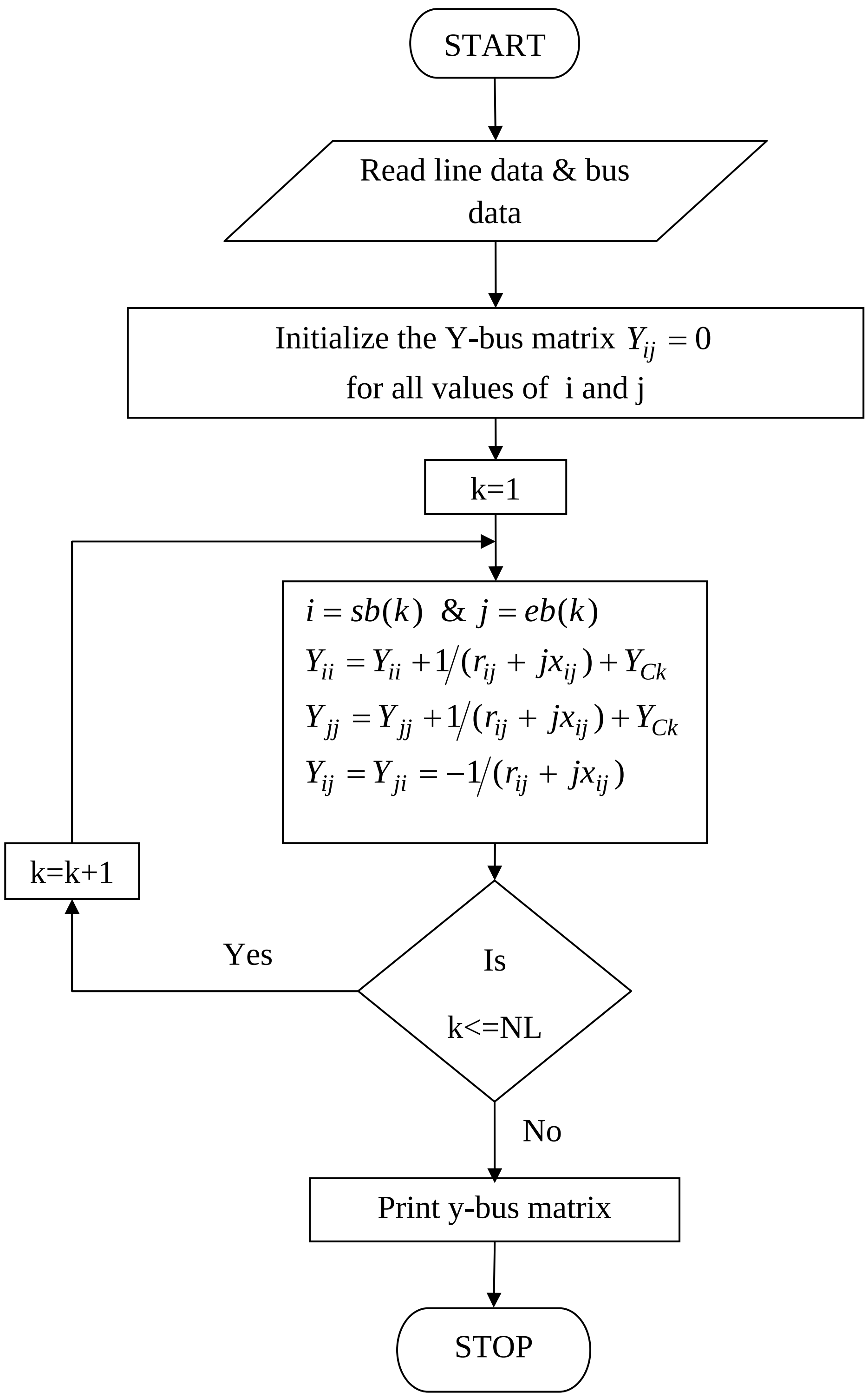
are the mutual admittances forming the off diagonal elements of the bus admittance matrix. For an n-bus system, the elements of the bus admittance matrix can be written down by inspection of the network as

Diagonal terms : $$Y_{ii} = y_{i0} + \sum_{\substack{k=1 \\ k \neq i}}^{n} y_{ik}$$

Off-diagonal terms : $$Y_{ij} = -y_{ij}$$

Note: This method of inspection is used only for those systems which do not contain ~~phase shifting network~~ mutually coupled elements.

**Flow Chart for Inspection Method**



Analytical method :- The Ybus matrix can be formed by analytical method for the systems with or without mutual coupling. The bus admittance matrix can be formed by using the ~~xx~~ relation

$$[Y] = [A][y][A]^T$$

where

$[Y]$ = bus admittance matrix, size $(nb \times nb)$

$[A]$ = reduced incidence matrix. (neglecting the ground node). size $(nb \times ne)$

$a_{ij} = 1$ ; if the $j^{th}$ element is incident at and oriented away from $i^{th}$ node

$a_{ij} = -1$ ; if the $j^{th}$ element is incident at and oriented towards the $i^{th}$ node.

$a_{ij} = 0$ ; if the $j^{th}$ element is not incident at the $i^{th}$ node.

$[y]$ = primitive admittance matrix $= [z]^{-1}$ size $(ne \times ne)$

ne : no. of elements.

nb : no. of buses.

$[z]$ = primitive impedance matrix, size $(ne \times ne)$

$z_{ii}$ : diagonal term of $[z]$, self impedance of $i^{th}$ element.

$z_{ij}$ : off-diagonal term of $[z]$, mutual impedance between the elements $i$ and $j$ ; if there is no mutual impedance, the value is zero.

Derivation of the formula :-

Let $[V]$ : bus voltages size $(nb \times 1)$

$[I]$ = vector of bus currents size $(nb \times 1)$

$[i]$ = vector of element currents size $(ne \times 1)$

$[v]$ = vector of element voltages size $(ne \times 1)$

The performance eqn in admittance form is given by

$$[I] = [Y][V] \quad \text{—(1)}$$

but $$[I] = [A][i] \quad \text{—(2)}$$

$$[v] = [A]^{tr}[V] \quad \text{—(3)}$$

$$[i] = [y][v] \quad \text{—(4)}$$

Sub. (3) & (4) in (2), we get

$$[I] = [A][y][A]^{tr}[V] \quad \text{—(5)}$$

Comparing (5) and (1), we can write

$$[Y] = [A][y][A]^{tr}.$$

<u>Algorithm</u>:-

1. Form bus incidence matrix $[A]$
2. Form primitive impedance matrix $[z]$
3. Compute primitive admittance matrix $[y]$ by inverting $[z]$
4. form $[Y]$ matrix by using
$$[Y] = [A][y][A]^{T}$$
5. Print the results.

**Problem**

A power system with 5 buses and 8 lines has the following data. Form the [Y] matrix by analytical method.

| Line | SB | EB | X |
|---|---|---|---|
| 1 | 1 | 4 | 0.6 |
| 2 | 5 | 1 | 0.2 |
| 3 | 2 | 3 | 0.2 |
| 4 | 2 | 4 | 0.4 |
| 5 | 2 | 5 | 0.2 |
| 6 | 3 | 4 | 0.4 |
| 7 | 1 | 0 | 0.25 |
| 8 | 2 | 0 | 1.25 |

**Mutual reactance**

betw. lines 2 & 5 : −0.15

1 & 4 : 0.3

1 & 3 : 0.1

**Soln.**

Primitive Impedance Matrix

$$[Z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0.6 & & 0.1 & 0.3 & & & & \\ & 0.2 & & & -0.15 & & & \\ 0.1 & & 0.2 & & & & & \\ 0.3 & & & 0.4 & & & & \\ & -0.15 & & & 0.2 & & & \\ & & & & & 0.4 & & \\ & & & & & & 0.25 & \\ & & & & & & & 1.25 \end{bmatrix} \end{array}$$

We have to invert the above matrix to find [Y].

**Simple method for finding the inverse:-**

① Choose the row which has least no. of off diagonal terms and form the submatrix. Then invert this submatrix.

**Row . 2**

$$\begin{array}{c} \\ 2 \\ 5 \end{array}\begin{array}{c} \begin{array}{cc} 2 & 5 \end{array} \\ \begin{bmatrix} 0.2 & -0.15 \\ -0.15 & 0.2 \end{bmatrix}^{-1} \end{array} = \begin{bmatrix} 11.4286 & 8.5714 \\ 8.5714 & 11.4286 \end{bmatrix}$$

Illy choose the next row

**Row . 1**

$$\begin{array}{c} \\ 1 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0 \\ 0.3 & 0 & 0.4 \end{bmatrix} \end{array} = \frac{1}{0.026}\begin{bmatrix} 0.08 & -0.04 & -0.06 \\ -0.04 & 0.15 & 0.03 \\ -0.06 & 0.03 & 0.11 \end{bmatrix}$$

$$
= \begin{bmatrix} 3.0769 & -1.5385 & -2.3077 \\ -1.5385 & 5.7692 & 1.1538 \\ -2.3077 & 1.1538 & 4.2308 \end{bmatrix}
$$

For those rows without off-diagonal terms, simply invert the diagonal values.

$$
[y] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
3.0769 & & -1.5385 & -2.3077 & & & & \\
 & 11.4286 & & & 8.5714 & & & \\
-1.5385 & & 5.7692 & 1.1538 & & & & \\
-2.3077 & & 1.1538 & 4.2308 & & & & \\
 & 8.5714 & & & 11.4286 & & & \\
 & & & & & 2.5 & & \\
 & & & & & & 4.0 & \\
 & & & & & & & 0.8
\end{bmatrix}
\end{array}
\quad (ne \times ne)
$$

<u>Formation of [A] matrix</u>

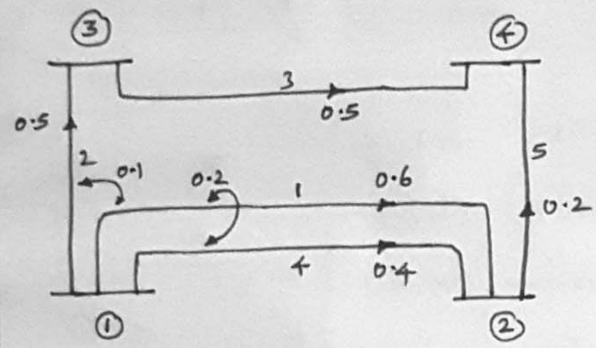
$$
[A] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
1 & -1 & & & & & 1 & \\
 & & 1 & 1 & 1 & & & 1 \\
 & & -1 & & & 1 & & \\
-1 & & & -1 & & -1 & & \\
 & 1 & & & -1 & & &
\end{bmatrix}
\end{array}
\quad (nb \times ne)
$$

$$
[A][y] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{bmatrix}
3.0769 & -11.4286 & -1.5385 & -2.3077 & -8.5714 & 0 & 4.0 & 0 \\
-3.8462 & 8.5714 & 6.923 & 5.3846 & 11.4286 & 0 & 0 & 0.8 \\
-1.5385 & 0 & -5.7692 & -1.1538 & 0 & 2.5 & 0 & 0 \\
-0.7692 & 0 & 0.3847 & -1.9231 & 0 & -2.5 & 0 & 0 \\
0 & 2.872 & 0 & 0 & -2.8572 & 0 & 0 & 0
\end{bmatrix}.
$$

$$[Y] = [A][y][A]^{tr} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 18.5055 & -12.4176 & 1.5385 & -0.7692 & -2.8572 \\ -12.4176 & 24.5362 & -6.923 & -1.5384 & -2.857 \\ -1.5385 & -6.923 & 8.2692 & 0.1923 & 0 \\ -0.7692 & -1.5384 & -2.8847 & 5.1923 & 0 \\ -2.872 & -2.8572 & 0 & 0 & 5.7292 \end{bmatrix} \end{array}$$

Step
pp. 61

Form the bus admittance matrix by the analytical method.




| Element no. | Self: Bus code | Self: Impedance | Mutual: Bus code | Mutual: Mutual Imp. |
|---|---|---|---|---|
| 1 | 1-2(1) | 0.6 | | |
| 2 | 1-3 | 0.5 | 1-2(1) | 0.1 |
| 3 | 3-4 | 0.5 | ~~1-2(1)~~ | |
| 4 | 1-2(2) | 0.4 | 1-2(1) | 0.2 |
| 5 | 2-4 | 0.2 | | |

Soln

$$[Z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & & 0.2 & \\ 0.1 & 0.5 & & & \\ & & 0.5 & & \\ 0.2 & & & 0.4 & \\ & & & & 0.2 \end{bmatrix} \end{array}$$

submatrix : $[X] = \begin{array}{c} \\ 1 \\ 2 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 4 \end{array} \\ \begin{bmatrix} 0.6 & 0.1 & 0.2 \\ 0.1 & 0.5 & 0 \\ 0.2 & 0 & 0.4 \end{bmatrix} \end{array}$ ; cofactor = $\begin{bmatrix} +(0.2) & +(0.04) & +(-0.1) \\ -(0.04) & +(0.2) & -(-0.02) \\ +(-0.1) & -(-0.02) & +(0.29) \end{bmatrix}$

$|\Delta| = 0.096.$   Inverse of $[X] = \begin{array}{c} 1 \\ 2 \\ 4 \end{array}\begin{bmatrix} 2.0839 & -0.4167 & -1.0417 \\ -0.4167 & 2.0833 & 0.2083 \\ -1.0417 & 0.2083 & 3.0208 \end{bmatrix}$

$$[y] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{bmatrix} 2.0839 & -0.4167 & 0 & -1.0417 & 0 \\ -0.4167 & 2.0833 & 0 & 0.2083 & 0 \\ 0 & 0 & 20.0 & 0 & 0 \\ -1.0417 & 0.2083 & 0 & 3.0208 & 0 \\ 0 & 0 & 0 & 0 & 5.0 \end{bmatrix}$$

$$
[A] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\left[\begin{array}{ccccc}
1 & 1 & 0 & 1 & 0 \\
-1 & 0 & 0 & -1 & 1 \\
0 & -1 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & -1
\end{array}\right]
\end{array}
$$

$$
[A][y] = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}
\left[\begin{array}{ccccc}
0.6255 & 1.8749 & 0.0 & 2.1874 & 0.0 \\
-1.0422 & 0.2084 & 0.0 & -1.9791 & 5.0 \\
0.4167 & -2.0833 & 20.0 & -0.2083 & 0.0 \\
0.0 & 0.0 & -20.0 & 0.0 & -5.0
\end{array}\right]
$$

$$
[A][y][A]^{tr} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}
\left[\begin{array}{cccc}
4.6878 & -2.8129 & -1.8749 & 0.0 \\
-2.8129 & 8.0213 & -0.2084 & -5.0 \\
-1.8749 & -0.2084 & 22.0833 & -20.0 \\
0.0 & -5.0 & -20.0 & 25.0
\end{array}\right]
$$

## Bus Impedance Matrix :

Bus impedance matrix [Z] is obtained by inverting the bus admittance matrix. It can also be formed by bus building algorithm.

- It is a square matrix
- It is a symmetric matrix.
- It is a non-singular matrix.
- It relates bus voltages and bus currents.

$$[V] = [Z][I]$$

Inversion process is a tedious process for forming [Z]. because the order of the matrix to be inverted is 'nb'. Therefore, this method cannot be used for larger networks. In addition, [Z] matrix can not be directly altered to reflect the changes (ie addition or removal of an element) in the network. It can be done by modifying the Ybus matrix and once again inverting it for the changes in the network.

An alternative method of forming the [Z] matrix is the bus building algorithm. It is a step by step procedure of forming [Z] matrix by adding one element at a time. In this method the [Z] matrix can be directly altered to reflect the changes in the network.

## Building Algorithm :

### Addition of a link with mutual coupling :-

$$Z_{bus} = Z_{bus}(0) - \frac{Z_{bus}(0)\ C_1\ C_1^T\ Z_{bus}(0)}{C_1^T\ Z_{bus}(0) \cdot C_1 + 1/y_{\alpha\alpha}} \quad 2 \text{ ①}$$

where

$$C_1 = K_1 + \frac{A_0\, y_{o\alpha}}{y_{\alpha\alpha}}$$

$K_1$ = submatrix of the [A] corr. to the added element. It indicates that how the new element is incident to the partial network.

$A_0$ = submatrix of [A] corr. to the elements of the partial network coupled to the new element. ~~without considering the new node~~.

$y_{\alpha\alpha}$ = self admittance of the element, obtained from the primitive admittance matrix formed by inverting the primitive impedance formed using only the coupled elements.

$[Y_{o\alpha}]$ = ~~vector~~ submatrix of primitive admittance matrix, corr. to the new element and the coupling elements.

<u>Addition of a link without mutual coupling</u> :-

$$Z_{bus} = Z_{bus(0)} - \frac{Z_{bus(0)} \cdot k_1 k_1^T Z_{bus(0)}}{k_1^T Z_{bus(0)} k_1 + Z_{\alpha\alpha}} \qquad 2\text{②}$$

<u>Addition of a branch with mutual coupling</u> :-

$$Z_{bus} = \begin{bmatrix} Z_{bus(0)} & Z_{bus(0)} \cdot C_2 \\ C_2^T Z_{bus(0)} & C_2^T Z_{bus(0)} \cdot C_2 + \frac{1}{y_{\alpha\alpha}} \end{bmatrix} \qquad 2\text{③}$$

where

$$C_2 = k_2 + \frac{A_o Y_{o\alpha}}{y_{\alpha\alpha}}.$$

$k_2$ = submatrix of A corresponding to the added element without considering the new node.

<u>Addition of a branch without mutual coupling</u> :-

$$Z_{bus} = \begin{bmatrix} Z_{bus(0)} & Z_{bus(0)} \cdot k_2 \\ k_2^T Z_{bus(0)} & k_2^T Z_{bus(0)} k_2 + Z_{\alpha\alpha} \end{bmatrix} \qquad 2\text{④}$$

<u>Steps</u> :-

① Identify the branches and links; and form the oriented graph.

② Choose the ref. node.

③ Start with the ref. node. Now Zbus matrix contains no elements

④ Add one element to the ref. bus. For this partial network, form the Zbus matrix using eqn. ④. The Zbus matrix corresponding to this partial network contains only one value.

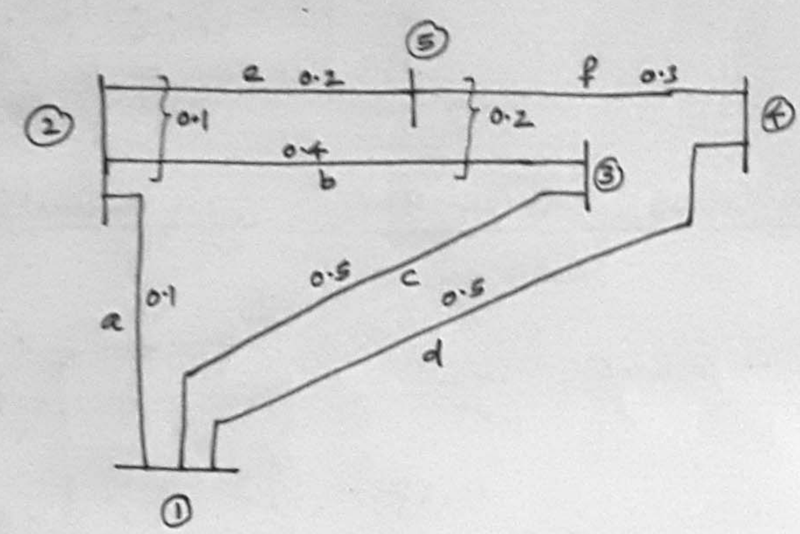
⑤ Add one more element, which may be a branch or a link to the partial ~~network using the appropriate eqn ① - ④, to the partial element~~ network and form the Zbus matrix using the appropriate eqn ① - ④

if the added element is a branch, the size of Zbus matrix will increase

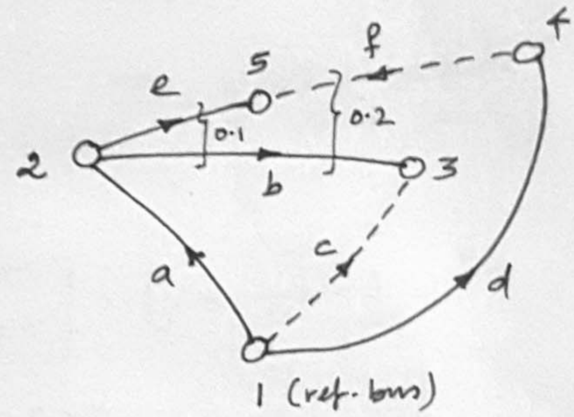
if the added element is a link, the size of Zbus matrix will not increase.

⑥ Repeat step ⑤, till all the elements are added one by one to the partial network.

Compute the bus impedance matrix for the network shown in Fig. Bus 1 can be taken as the reference bus, since there are no shunt connections to ground in this case.




Soln:-

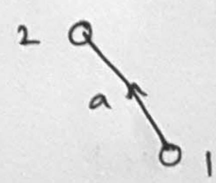



① Draw the oriented graph by identifying the branches and links.

② Choose the ref. bus. if there are shunt connections to the ground, then ground is taken as the ref. node, otherwise choose any other bus as the reference node. In this example, bus 1 is taken as the reference node.

③ element - a : Addition of a branch :

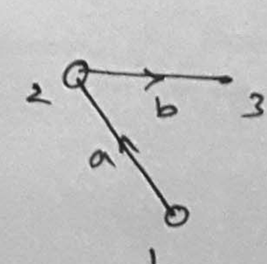
$$Z_{bus} = \begin{array}{c|c} & 2 \\ 2 & \begin{bmatrix} 0.1 \end{bmatrix} \end{array}$$




④ element - b : Addition of a branch w/o mutual coupling :

$$Z_{bus} = \begin{bmatrix} Z_{bus}(0) & Z_{bus}(0)\, K_2 \\ K_2^T Z_{bus}(0) & K_2^T Z_{bus}(0)\, K_2 + Z_{\alpha\alpha} \end{bmatrix}$$

$$[A] = \begin{array}{c|c} & \begin{matrix} a & b \end{matrix} \\ \begin{matrix} 2 \\ 3 \end{matrix} & \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \end{array} \leftarrow K_2$$



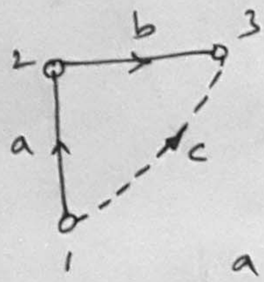

$$[z] = \begin{array}{c|c} & \begin{matrix} a & b \end{matrix} \\ \begin{matrix} a \\ b \end{matrix} & \begin{bmatrix} 0.1 & 0 \\ 0 & \boxed{0.4} \end{bmatrix} \end{array}$$

— $Z_{\alpha\alpha}$ : impedance of the added branch.

$k_2 = [1]$ ; $Z_{dd} = [0.4]$ ; $Z_{bus}(0) = [0.1]$

$$Z_{bus} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1+0.4 \end{bmatrix} = \begin{array}{c|cc} & 2 & 3 \\ 2 & \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} \\ 3 & & \end{array}$$

⑤ element - c : Addition of a link w/o mutual coupling



$$Z_{bus} = Z_{bus}(0) - \frac{Z_{bus}(0) \cdot k_1 k_1^T Z_{bus}(0)}{k_1^T Z_{bus}(0) \cdot k_1 + Z_{dd}}$$

$$[A] = \begin{array}{c|ccc} & a & b & c \\ 2 & -1 & 1 & 0 \\ 3 & 0 & -1 & -1 \end{array} \quad \leftarrow k_1$$

$Z_{dd}$ = self impedance of the line $\} = 0.5$

$$k_1^T Z_{bus} = [0 \ -1] \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} = [-0.1 \ -0.5]$$

$$k_1^T Z_{bus} k_1 = [-0.1 \ -0.5] \begin{bmatrix} 0 \\ -1 \end{bmatrix} = [0.5]$$

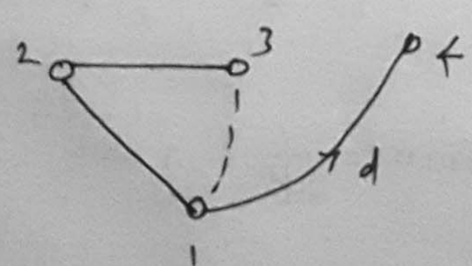
$$Z_{bus} \cdot k_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.1 \\ -0.5 \end{bmatrix}$$

$$Z_{bus} k_1 k_1^T Z_{bus} = \begin{bmatrix} -0.1 \\ -0.5 \end{bmatrix} [-0.1 \ -0.5] = \begin{bmatrix} 0.01 & 0.05 \\ 0.05 & 0.25 \end{bmatrix}$$

$$k_1^T Z_{bus} k_1 + Z_{dd} = 0.5 + 0.5 = 1.0$$

$$Z_{bus} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.5 \end{bmatrix} - \left( \begin{bmatrix} 0.01 & 0.05 \\ 0.05 & 0.25 \end{bmatrix} \div 1.0 \right) = \begin{bmatrix} 0.09 & 0.05 \\ 0.05 & 0.25 \end{bmatrix}$$

⑥ element - d : Addition of a branch w/o mutual coupling



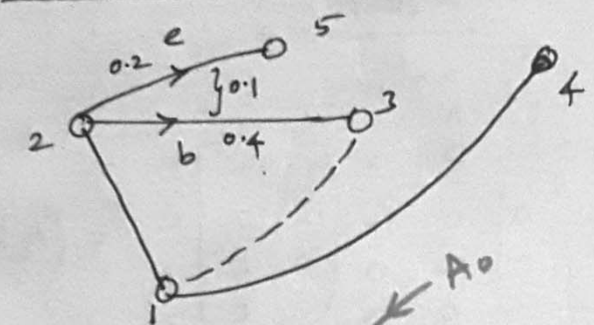
$$Z_{bus} = \begin{bmatrix} Z_{bus}(0) & Z_{bus}(0)\, k_2 \\ k_2^T Z_{bus}(0) & k_2^T Z_{bus}(0)\, k_2 + Z_{dd} \end{bmatrix}$$

$$[A] = \begin{array}{c|cccc} 2 & -1 & 1 & 0 & \boxed{0} \\ 3 & 0 & -1 & -1 & \boxed{0} \\ 4 & 0 & 0 & 0 & -1 \end{array} \leftarrow k_2$$

$Z_{dd} = [0.5]$

$$Z_{bus} = \left[\begin{array}{cc|c} 0.09 & 0.05 & 0.0 \\ 0.05 & 0.25 & 0.0 \\ \hline 0.0 & 0.0 & 0.5 \end{array}\right]$$

(7) element - e : Addition of a branch with mutual coupling.




$$Z_{bus} = \begin{bmatrix} Z_{bus}(0) & Z_{bus}(0) \cdot C_2 \\ C_2^T Z_{bus}(0) & C_2^T Z_{bus}(0) \cdot C_2 + \frac{1}{Y_{\alpha\alpha}} \end{bmatrix}$$

$A_0$

$$[A] = \begin{array}{c|c} & a \quad b \quad c \quad d \quad e \\ \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \left[\begin{array}{cccc|c} -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ \hline 0 & 0 & 0 & 0 & -1 \end{array}\right] \end{array} \quad K_2$$

$$C_2 = K_2 + \frac{A_0 Y_{o\alpha}}{Y_{\alpha\alpha}}$$

$$[K_2] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ; \quad [A_0] = \begin{bmatrix} \text{col. vector of the coupled element } b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$[z_c] = \begin{array}{c|c} & b \quad\quad e \\ \begin{matrix} b \\ e \end{matrix} & \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \end{array} \quad |\Delta| = 0.07 \quad ; \quad [y_c] = [z_c]^{-1} = \begin{array}{c|c} & b \quad\quad e \\ \begin{matrix} b \\ e \end{matrix} & \begin{bmatrix} 2.857 & -1.4286 \\ -1.4286 & 5.714 \end{bmatrix} \end{array}$$

$Y_{o\alpha}$

$Y_{\alpha\alpha}$

$$C_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \frac{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} [-1.4286]}{[5.714]} = \begin{bmatrix} 0.75 \\ 0.25 \\ 0.0 \end{bmatrix}$$

$$C_2^T Z_{bus} = \begin{bmatrix} 0.75 & 0.25 & 0.0 \end{bmatrix} \begin{bmatrix} 0.09 & 0.05 & 0.0 \\ 0.05 & 0.25 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.08 & 0.1 & 0.0 \end{bmatrix}$$

$$C_2^T Z_{bus} C_2 = \begin{bmatrix} 0.08 & 0.1 & 0.0 \end{bmatrix} \begin{bmatrix} 0.75 \\ 0.25 \\ 0.0 \end{bmatrix} = [0.085]$$

$$C_2^T Z_{bus} C_2 + \frac{1}{Y_{\alpha\alpha}} = 0.085 + \frac{1}{5.714} = 0.26$$

$$[Z_{bus}] = \begin{bmatrix} 0.09 & 0.05 & 0.0 & 0.08 \\ 0.05 & 0.25 & 0.0 & 0.1 \\ 0.0 & 0.0 & 0.5 & 0.0 \\ 0.08 & 0.1 & 0.0 & 0.26 \end{bmatrix}$$

⑧ element f : Addition of a link with mutual coupling :

$$Z_{bus} = Z_{bus}(0) - \frac{Z_{bus}(0) \cdot C_1 \cdot C_1^T \, Z_{bus}(0)}{C_1^T \, Z_{bus}(0) \, C_1 + (1/y_{\alpha\alpha})}.$$

$$C_1 = K_1 + \frac{A_0 y_0 \alpha}{y_{\alpha\alpha}}$$

$$[A] = \begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 0 & -1 & -1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & -1 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 & -1 & -1 \end{array} \quad \leftarrow K_1$$

$A_0$

$$[A_0] = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} ; \quad [K_1] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} : \quad [\mathcal{Z}_c] = \begin{array}{c|ccc} & b & e & f \\ \hline b & 0.4 & 0.1 & -0.2 \\ e & 0.1 & 0.2 & 0 \\ f & -0.2 & 0 & 0.3 \end{array}$$

$$\text{cofactor of } [\mathcal{Z}_c] = \begin{array}{c|ccc} & b & e & f \\ \hline b & +(0.06) & -(0.03) & +(0.04) \\ e & -(0.03) & +(+0.08) & -(0.02) \\ f & +(0.04) & -(0.02) & +(0.07) \end{array} \qquad |\Delta| = 0.0130$$

$$[y_c] = \begin{array}{c|ccc} & b & e & f \\ \hline b & 4.6154 & -2.3077 & 3.0769 \\ e & -2.3077 & 6.9538 & -1.5385 \\ f & 3.0769 & -1.5385 & 5.3846 \end{array}$$

$\leftarrow Y_{0\alpha}$

$\leftarrow y_{\alpha\alpha}$

$$[C_1] = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3.0769 \\ -1.5385 \end{bmatrix} \Big/ 5.3846$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.2857 \\ -0.5714 \\ 0.0 \\ 0.2857 \end{bmatrix} = \begin{bmatrix} 0.2857 \\ -0.5714 \\ 1.0 \\ -0.7143 \end{bmatrix}$$

$$C_1^T Z_{bus} = \begin{bmatrix} -0.0600 & -0.2000 & 0.5 & -0.22 \end{bmatrix}$$

$$C_1^T Z_{bus} C_1 = 0.7578$$

$$-\text{"}- + 1/y_{\alpha\alpha} = 0.7578 + 1/5.3846 = 0.9435.$$

$$[C_1 \; C_1^T \; Z_{bus}] = \begin{bmatrix} -0.0171 & -0.0571 & 0.1429 & -0.0629 \\ 0.0343 & 0.1143 & -0.2857 & 0.4082 \\ -0.0600 & -0.2000 & 0.5 & -0.2200 \\ 0.0429 & 0.8429 & -0.3572 & 0.1571 \end{bmatrix}$$

$$[Z_{bus}][C_1 C_1^T Z_{bus}] = \begin{bmatrix} 0.0036 & 0.012 & -0.03 & 0.0273 \\ 0.012 & 0.04 & -0.1 & 0.1146 \\ -0.03 & -0.1 & 0.25 & -0.11 \\ 0.0132 & 0.044 & -0.11 & 0.0766 \end{bmatrix}$$

$$Z_{bus} = Z_{bus} - [\; -\text{''}- \;] / 0.9435$$

$$=$$

## Modifications to an existing network:

### Removal of a link with mutual coupling:

$$Z_{bus} = Z_{bus}(0) - \frac{Z_{bus}(0)\, C_1 C_1^T\, Z_{bus}(0)}{C_1^T Z_{bus}(0)\, C_1 - (1/Y_{\alpha\alpha})}$$

the sign is the only change compared to the eqn for addition of a link.

where

$$C_1 = K_1 + \frac{A_0 Y_{0\alpha}}{Y_{\alpha\alpha}}$$

### Removal of a link with no mutual coupling:

$$Z_{bus} = Z_{bus}(0) - \frac{Z_{bus}(0)\, K_1 K_1^T\, Z_{bus}(0)}{K_1^T Z_{bus}(0)\, K_1 - Z_{\alpha\alpha}}$$

Removal of a radial line :- When an element corresponding to a radial line is removed, one bus get isolated and the number of buses in the network is reduced by one. It can be done by deleting the row and column corresponding to the isolated bus in the original bus impedance matrix.

If the isolated bus is the reference bus itself, then the bus impedance matrix of the new network is indefinite.

Parameter changes :- When the parameter of an element is changed, the bus impedance matrix can be modified by simultaneously removing the element with the old parameter and adding an element with the revised parameter.

# UNIT-III SPARSITY TECHNIQUES

## INTRODUCTION

- Sparsity is the condition of not having enough of something.
- If a matrix contains less number of non-zero elements, then that matrix is considered as sparse matrix. In power systems, most of the matrices like Ybus matrix and Jacobian matrix are sparse matrices.
- Sparsity technique is a programming technique is a digital programming technique by which sparse matrices are stored in a compact form in computer memory.
- Only non-zero elements are stored and calculations are done on non-zero values, thereby not only reducing the computer memory requirement but also reducing the computation time.
- Most the software programs use sparsity techniques effectively in solving very large problems like power flow of Indian Power System.

## SPARSITY TECHNIQUES

1. Compact Storage Scheme
2. LU Factorization
3. Optimal Ordering

## COMPACT STORAGE SCHEME

While storing non-zero elements of sparse matrices in computer memory, a systematic procedure must be adapted so that the non-zero element can be accessed, altered, included or removed. To handle sparse matrices, two methods are popularly used.

- Entry-Row-Column Method
- Chained Data-Structure Method

### Entry-Row Column Method

- Consider a sparse matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

- The above matrix can be stored in compact form as follows:

| STO | RN | CN |
|---|---|---|
| 1 | 1 | 2 |
| 3 | 2 | 1 |
| 2 | 3 | 3 |

where
STO : Stored Non-Zero Values
RN : Row Number
CN : Column Number

- It is very clear from the above example that there are three linear vectors to store non-zero values.
- These three vectors contain all the data present in the original [A] matrix.
- This is the simplest method but it has some drawbacks.
- The main drawback is that data retrieval is not so fast.
- This method is not followed in practice.

### Chained Data-Structure Method

- Consider a sparse matrix $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

- The above matrix can be stored in compact form as follows:

| | STO | CN | . . . . | N-First | |
|---|---|---|---|---|---|
| *(1) -->* | **1** | **1** | | **1** | *--> First row starts from array index (1)* |
| *(2) -->* | **1** | **4** | | **3** | *--> Second row starts from array index (3)* |
| *(3) -->* | **4** | **1** | | **5** | *--> Third row starts from array index (5)* |
| *(4) -->* | **3** | **2** | | **6** | *--> Forth row starts from array index (6)* |
| *(5) -->* | **2** | **4** | | **7** | |
| *(6) -->* | **1** | **3** | | | |
| *(7) -->* | | | | | |
| *(8) -->* | | | | | |

The value-1 in NX vector indicates that there are some more values in the respective row.
If NX=0, there are no more non-zero values in the respective row.

- This method replaces the RN vector by RFirst vector, whose size equals only the number of rows in the given matrix, which further reduces the memory requirement.
- The numbers in the RFirst arrays indicate the index numbers of STO/CN arrays and represent where the a row starts in STO/CN arrays.
- This method is widely used in all practical applications.

**LU FACTROIZATION OR TRIANGULAR FACTORIZATION**

* This is one of the sparsity techniques and serves the purpose of reducing the calculation time.

$$[A]_{n\times n}\,[x]_{n\times 1} = [B]_{n\times 1} \longrightarrow (1)$$

* One way of solving for x is to invert [A] matrix and then multiply it by [B]

$$[x] = [A]^{-1}[B]$$

* This Procedure is not recommended because,

(i) Time taken will be more.
(ii) Round off errors will be there.
(iii) $[A]^{-1}$ will be a full matrix; Particularly in case of $[Y_{BUS}]^{-1}$, so storage requirement will be more.

* For these reasons, we adopt LU factorisation by which all the above mentioned drawbacks are removed.

* According to this method, Square matrix [A] is factorised into 2 matrices [L] and [U] such that,

$$[A] = [L][U]$$

* L is defined as follows.

$$[L] = \begin{bmatrix} l_{11} & 0 & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & 0 & \cdots & 0 \\ l_{31} & l_{32} & l_{33} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & \cdots & l_{nn} \end{bmatrix}$$

* U is defined as follows.

$$[U] = \begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & \cdots & u_{1n} \\ 0 & 1 & u_{23} & \cdots & \cdots & u_{2n} \\ 0 & 0 & 1 & u_{34} & \cdots & u_{3n} \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

where $[L] \Rightarrow$ Lower Triangular Matrix.

$[U] \Rightarrow$ Upper Triangular Matrix.

* Eqn. (1) gets modified as follows:

$$[L][U][x] = [B]$$

$$[L][K] = [B]$$

* Find [k] by Forward substitution by solving $[L][k] = [B]$

* Solve for [x] by Backward substitution by solving $[U][x] = [k]$.

* Thus, by backward and forward substitution, we get the answer very quickly.

* Let us factorise [A] into [L] & [U] matrices such that $[A] = [L][U]$.

* The elements of [L] and [U] matrices may be determined by using the formulae,

$$u_{ij} = \frac{\left(A_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj}\right)}{l_{ii}} \quad i<j.$$

$$l_{ij} = \left(A_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj}\right) \quad i \geq j.$$

EXAMPLE :

Solve the following Matrix Equation by applying LU factorization

$$\begin{bmatrix} 1 & 0 & -2 & 3 & 2 \\ 0 & -2 & 3 & 1 & 0 \\ 3 & 1 & 0 & -2 & 1 \\ -2 & 3 & 4 & 0 & 1 \\ 2 & 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 2 \\ 21 \\ 19 \end{bmatrix}$$

Solution :

$$AX = B.$$

$$LUX = B$$

$$UX = K$$

$$LK = B$$

$$\boxed{\text{obtain } K}$$

$$UX = k$$

$$\boxed{\text{obtain } x}$$

Begin with [L] matrix

Consider first column of [L]

$L_{11}, L_{21}, L_{31}, L_{41}, L_{51}$

$$l_{ij} = a_{ij} - \sum_{k=1}^{j-1} l_{ik} U_{kj}.$$

$$L_{11} = a_{11} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj} = a_{11} - 0 = a_{11} = 1$$

$$l_{21} = a_{21} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj} = a_{21} - 0 = a_{21} = 0$$

$$l_{31} = a_{31} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj} = a_{31} - 0 = a_{31} = 3.$$

$$l_{41} = a_{41} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj} = a_{41} - 0 = a_{41} = -2$$

$$l_{51} = a_{51} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj} = a_{51} - 0 = a_{51} = 2$$

Consider first row of [U]

$U_{11}, U_{12}, U_{13}, U_{14}, U_{15}$

$$U_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} l_{ik} U_{kj}}{l_{ii}}$$

$$U_{11} = \frac{a_{11} - \sum_{k=1}^{1-1=0} l_{ik} U_{kj}}{l_{11}} = \frac{a_{11} - 0}{l_{11}} = \frac{a_{11}}{l_{11}} = \frac{1}{1} = 1.$$

$$U_{12} = \frac{a_{12} - 0}{l_{11}} = \frac{a_{12}}{l_{11}} = \frac{0}{1} = 0.$$

$$U_{13} = \frac{a_{13} - 0}{l_{11}} = \frac{a_{13}}{l_{11}} = \frac{-2}{1} = -2.$$

$$U_{14} = \frac{a_{14} - 0}{l_{11}} = \frac{a_{14}}{l_{11}} = \frac{3}{1} = 3$$

$$u_{15} = \frac{a_{15} - 0}{l_{11}} = \frac{a_{15}}{l_{11}} = \frac{2}{1} = 2.$$

**SECOND COLUMN OF [L] MATRIX:**

$$l_{22} = a_{22} - \sum_{k=1}^{2-1} l_{ik} U_{kj} = a_{22} - l_{21} U_{12} = -2 - (0)(0) = -2$$

$$l_{32} = a_{32} - \sum_{k=1}^{2-1} l_{3k} U_{k2} = a_{32} - l_{31} U_{12} = 1 - (3)(0) = 1.$$

$$l_{42} = a_{42} - \sum_{k=1}^{2-1} l_{4k} U_{k2} = a_{42} - l_{41} U_{12} = 3 - (-2)(0) = 3$$

$$l_{52} = a_{52} - \sum_{k=1}^{2-1} l_{5k} U_{k2} = a_{52} - l_{51} U_{12} = 0 - (2)(0) = 0$$

**SECOND ROW OF [U] MATRIX:**

$$u_{22} = \frac{a_{22} - \sum_{k=1}^{1} l_{2k} U_{k2}}{l_{22}} = \frac{a_{22} - l_{21} U_{12}}{l_{22}} = \frac{-2}{-2} = 1$$

$$u_{23} = \frac{a_{23} - \sum_{k=1}^{1} l_{2k} U_{k3}}{l_{22}} = \frac{a_{23} - l_{21} U_{13}}{l_{22}} = \frac{3 - (0)(-2)}{-2} = \frac{-3}{2}$$

$$U_{24} = \frac{a_{24} - \sum_{k=1}^{1} l_{2k} U_{k4}}{l_{22}} = \frac{a_{24} - l_{21} U_{14}}{l_{22}} = \frac{1 - (0)(3)}{-2} = \frac{-1}{2}$$

$$U_{25} = \frac{a_{25} - \sum_{k=1}^{1} l_{2k} U_{k5}}{l_{22}} = \frac{a_{25} - l_{21} U_{15}}{l_{22}} = \frac{0 - (0)(2)}{-2} = 0.$$

THIRD COLUMN OF [L] MATRIX:

$$l_{33} = a_{33} - \sum_{k=1}^{2} l_{3k} U_{k3} = 0 - [l_{31} U_{13} + l_{32} U_{23}]$$

$$= 0 - \left[3(-2) + (1)\left(\frac{-3}{2}\right)\right]$$

$$= 6 + \frac{3}{2}$$

$$l_{33} = 15/2$$

$$l_{43} = a_{43} - \sum_{k=1}^{2} l_{4k} U_{k3} = 4 - [l_{41} U_{13} + l_{42} U_{23}]$$

$$= 4 - [(-2)(-2) + (3)(-3/2)]$$

$$= 4 - \left[4 - \frac{9}{2}\right]$$

$$l_{43} = \frac{9}{2}$$

$$l_{53} = a_{53} - \sum_{k=1}^{2} l_{5k} U_{k3} = a_{53} - [l_{51} U_{13} + l_{52} U_{23}]$$

$$= 1 - \left[(2)(-2) + (0)\left(\frac{-3}{2}\right)\right]$$

$$= 1 + 4$$

$$l_{53} = 5$$

THIRD ROW OF [U] MATRIX:

$$U_{33} = 1$$

$$U_{34} = \frac{a_{34} - \sum_{k=1}^{2} l_{3k} U_{k4}}{l_{33}} = \frac{a_{34} - (l_{31} U_{14} + l_{32} U_{24})}{l_{33}}$$

$$= \frac{-2 - [(3)(3) + (1)(-0.5)]}{7.5}$$

$$= \frac{-2 - [9 - 0.5]}{7.5}$$

$$U_{34} = -1.4$$

$$U_{35} = \frac{a_{35} - \sum_{k=1}^{2} l_{3k} U_{k5}}{l_{33}} = \frac{a_{35} - (l_{31} U_{15} + l_{32} U_{25})}{l_{33}}$$

$$= \frac{1 - [(3)(2) + (1)(0)]}{7.5}$$

$$U_{35} = -0.666$$

FOURTH COLUMN OF [l] Matrix :

$$l_{44} = a_{44} - \sum_{k=1}^{4-1} l_{4k} U_{k4} = a_{44} - [l_{41} U_{14} + l_{42} U_{24} + l_{43} U_{34}]$$

$$= 0 - [(-2)(3) + (3)(-0.5) + (4.5)(-1.4)]$$

$$= 0 - [-6 - 1.5 - 6.3]$$

$$l_{44} = 13.8$$

$$l_{54} = a_{54} - \sum_{k=1}^{4-1} l_{5k} U_{k4} = a_{54} - [l_{51} U_{14} + l_{52} U_{24} + l_{53} U_{34}]$$

$$= 1 - [(2)(3) + (0)(-0.5) + (5)(-1.4)]$$

FOURTH ROW OF [U] :

$$l_{54} = 2$$

$$U_{45} = \frac{a_{45} - \sum_{k=1}^{3} l_{4k} U_{k5}}{l_{44}} = \frac{1 - [l_{41} U_{15} + l_{42} U_{25} + l_{43} U_{35}]}{l_{44}}$$

$$= \frac{1 - [(-2)(2) + (3)(0) + (4.5)(-0.666)]}{13.8}$$

$$U_{45} = 0.5797$$

FIFTH COLUMN OF [L] :

$$l_{55} = a_{55} - [l_{51} U_{15} + l_{52} U_{25} + l_{53} U_{35} + l_{54} U_{45}]$$

$$= 2 - [(2)(2) + (0)(0) + (5)(-0.666) + (2)(0.5797)]$$

$$l_{55} = 0.1739.$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 3 & 1 & 7.5 & 0 & 0 \\ -2 & 3 & 4.5 & 13.8 & 0 \\ 2 & 0 & 5 & 2 & 0.1739 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 & -2 & 3 & 2 \\ 0 & 1 & -1.5 & -0.5 & 0 \\ 0 & 0 & 1 & -1.4 & -0.666 \\ 0 & 0 & 0 & 1 & 0.5797 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

FORWARD SUBSTITUTION :

$$LK = B$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 3 & 1 & 7.5 & 0 & 0 \\ -2 & 3 & 4.5 & 13.8 & 0 \\ 2 & 0 & 5 & 2 & 0.1739 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 2 \\ 21 \\ 19 \end{bmatrix}$$

$\Rightarrow \boxed{k_1 = 17}$

$\Rightarrow -2k_2 = 19$

$k_2 = -19/2 = -9.5 \quad \boxed{k_2 = -9.5}$

$\Rightarrow 3k_1 + k_2 + 7.5k_3 = 2$

$3(17) + (-9.5) + 7.5k_3 = 2$

$\boxed{k_3 = -5.267}$

$\Rightarrow -2k_1 + 3k_2 + 4.5k_3 + 13.8k_4 = 21$

$-2(17) + 3(-9.5) + 4.5(-5.267) + 13.8k_4 = 21$

$\boxed{k_4 = 7.768}$

$$\Rightarrow \quad 2k_1 + 5k_3 + 2k_4 + 0.1739k_5 = 19.$$

$$2(17) + 5(-5.267) + 2(7.768) + 0.1739k_5 = 19$$

$$\boxed{k_5 = -24.157}$$

$k_1 = 17$

$k_2 = -9.5$

$k_3 = -5.267$

$k_4 = 7.768$

$k_5 = -24.157$

BACKWARD SUBSTITUTION:

$$UX = K.$$

$$\begin{bmatrix} 1 & 0 & -2 & 3 & 2 \\ 0 & 1 & -1.5 & -0.5 & 0 \\ 0 & 0 & 1 & -1.4 & -0.666 \\ 0 & 0 & 0 & 1 & 0.5797 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 17 \\ -9.5 \\ -5.267 \\ 7.768 \\ -24.157 \end{bmatrix}$$

$$\Rightarrow \quad x_5 = -24.157$$

$$\Rightarrow \quad x_4 + 0.5797x_5 = 7.768$$

$$x_4 = 21.768$$

$$\Rightarrow \quad x_3 - 1.4x_4 - 0.666x_5 = -5.267$$

$$x_3 = 9.12$$

$$\Rightarrow \quad x_2 - 1.5x_3 - 0.5x_4 + 0x_5 = -9.5$$

$$x_2 = 15.06$$

$$\Rightarrow \quad x_1 + 0x_2 - 2x_3 + 3x_4 + 2x_5 = 17$$

$$x_1 = 18.25$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 18.25 \\ 15.06 \\ 9.12 \\ 21.768 \\ -24.157 \end{bmatrix}$$

## [3] OPTIMAL ORDERING:

* The Optimal Ordering is a process to obtain a new order of rows and columns to be eliminated in the factorization process in such a way to reduce the number of fill-ins.

* In other words, Optimal Ordering refers to renumbering the matrix order so that fill-ins are reduced.

* As the number of non-zero values to be stored in Computer is minimised and hence, the computation time is reduced. Computer Memory is also saved.

### TINNEY'S SCHEMES FOR NEAR OPTIMAL ORDERING:

**Method ①:**

* The factorisation of a row or a column with minimum number of non-zero entries will generate minimum number of fill-ins & vice-versa.

* In this method, number of non-zero entries at each row & column are counted.

* Row (or column) with minimum no. of non-zeros is considered as first row (or column).

* The Row (or Column) with next few non-zeros is considered as second row (or column) and so on.

* That is, Rows & Columns are arranged in ascending order based on number of non-zero entries.

* LU factorisation is carried out based on this new order.

Method ② (Tinney - Walker Method):

* Choose the row with minimum non-zero entries as first row.

* similarly, select the column with minimum non-zero entries.

* Apply LU factorization (or) simulate the factorization process on the selected row & column. This may create new fill-ins.

* Omitting the rows & columns, that are already processed, once again choose the row & column with minimum no. of non-zero entries accounting the new fill-ins created by factorization process.

* This Process may be repeated till all the rows & columns are processed.

Method ③:

* Choose the row and column that will generate minimum fill-ins.

* Simulate the factorization process in order to find the fill-ins on each row (or column).

* Once again, Repeat the simulation process for remaining rows, taking into account the fill-ins already generated.

* This Procedure is followed till all rows & columns are selected.

**Procedure for Tinney & Walker Method:**

* To identify extra fill-ins on account of LU factorisation, we have shortcut procedure or Thumb Rule.

* Extra-fill in should be even number. As the given matrix is symmetrical matrix, if extra fill-ins come into $y_{ij}$, then one more at $y_{ji}$ (i.e.,) for example, $y_{13} = y_{31}$

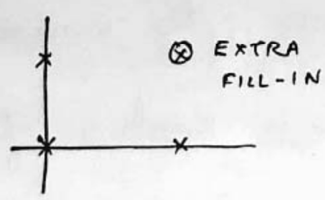
* We have to see the minimum number of non-zeros (x) (or) maximum number of zeros (blanks) and take that row as first row. Take the column as first column. Cut that row & column by a straight line.

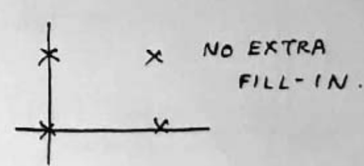
* Try to form all possible square or rectangular form from the intersection point of that row & column.

* Verify all corners of square or rectangle possess the non-zero element.

* If not, put extra fill-ins by ⊗ symbol in a square or rectangular corners which are not having non-zero element.

* If all corners of square or rectangle possess non-zero elements, then no need to fill up any extra fill-ins.




* While counting non-zero elements, include extra fill-ins also.

| | ① | ② | ③ | ④ | | |
|---|---|---|---|---|---|---|
| e.g.:- (i) | × | ✱ | ✱ | × | ⇒ | 2 Non-zeros |
| (ii) | × | ⊗ | × | × | ⇒ | 4 Non-zeros. |

* New fill-ins do not exist on cut line.

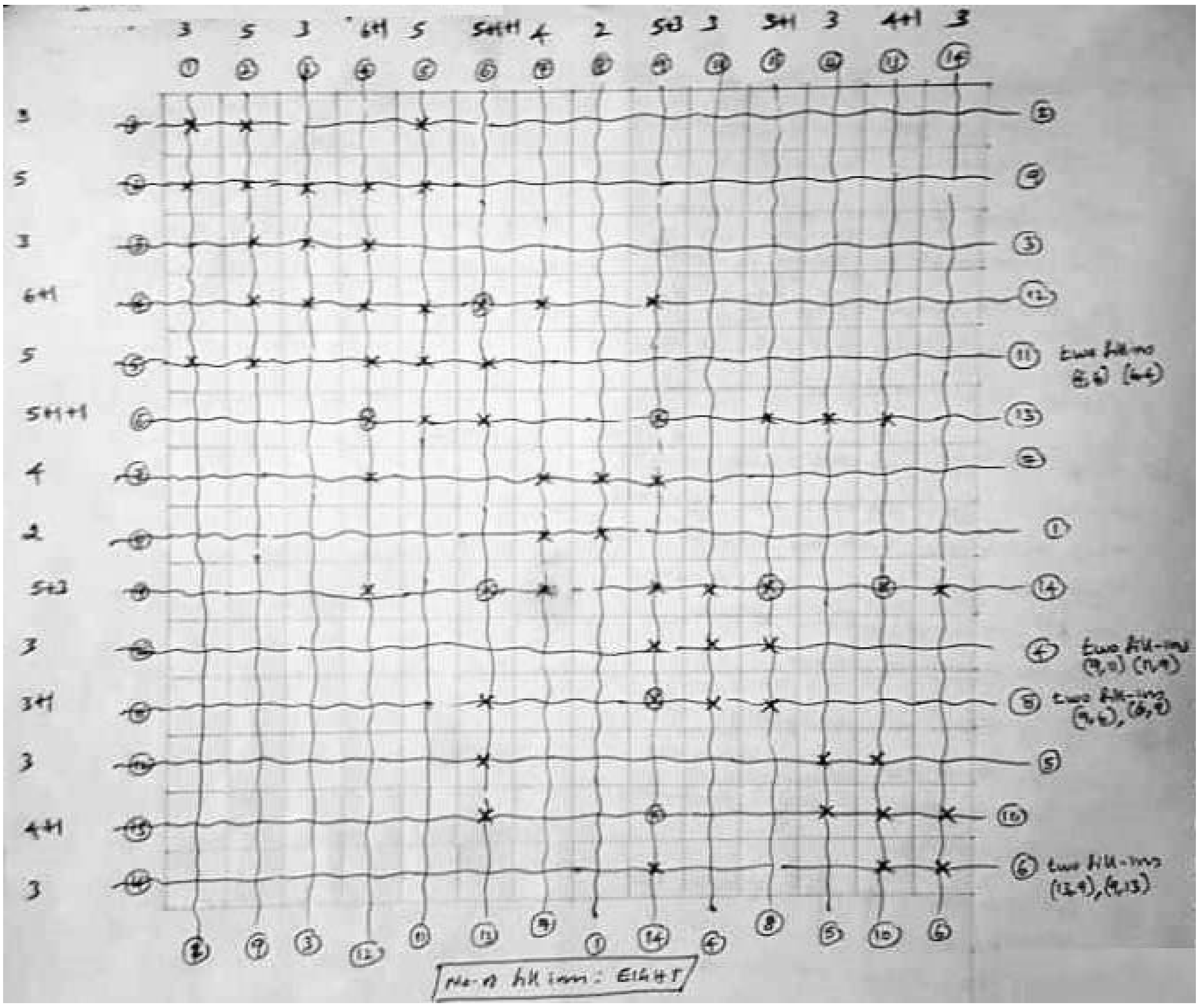
* The Procedure gets repeated until all rows & columns are taken into account.

* Count the total no. of extra fill-ins at end & it is always very less than no. of extra fill-ins without optimal ordering.

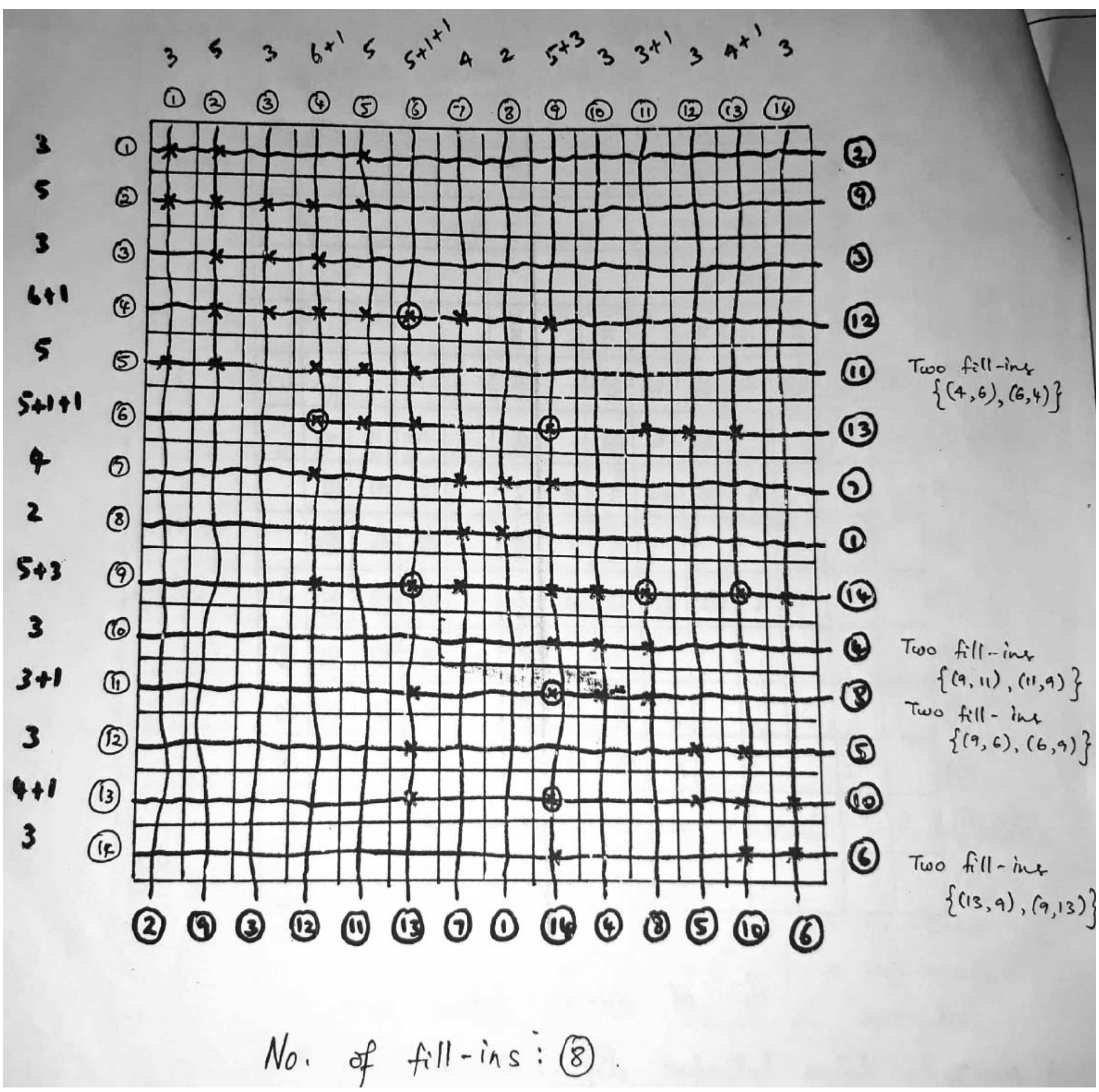
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | -- | 1 | X | X | | | X | | | | | | | | | |
| 5 | -- | 2 | X | X | X | X | X | | | | | | | | | |
| 3 | -- | 3 | | X | X | X | | | | | | | | | | |
| 6 | -- | 4 | | X | X | X | X | | X | | X | | | | | |
| 5 | -- | 5 | X | X | | X | X | X | | | | | | | | |
| 5 | -- | 6 | | | | | X | X | | | | | X | X | X | |
| 4 | -- | 7 | | | | X | | | X | X | X | | | | | |
| 2 | -- | 8 | | | | | | | X | X | | | | | | |
| 5 | -- | 9 | | | | X | | | X | | X | X | | | | X |
| 3 | -- | 10 | | | | | | | | | X | X | X | | | |
| 3 | -- | 11 | | | | | | X | | | | X | X | | | |
| 3 | -- | 12 | | | | | | X | | | | | | X | X | |
| 4 | -- | 13 | | | | | | X | | | | | | X | X | X |
| 3 | -- | 14 | | | | | | | | | X | | | | X | X |

**PROBLEM: Perform Optimal Ordering by Tinney-Walker Method-2 for the following Matrix, where X represents non-zero elements.**


| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | -- | 1 | X | X | | | X | | | | | | | | | |
| 5 | -- | 2 | X | X | X | X | X | | | | | | | | | |
| 3 | -- | 3 | | X | X | X | | | | | | | | | | |
| 6 | -- | 4 | | X | X | X | X | | X | | X | | | | | |
| 5 | -- | 5 | X | X | | X | X | X | | | | | | | | |
| 5 | -- | 6 | | | | | X | X | | | | | X | X | X | |
| 4 | -- | 7 | | | | X | | | X | X | X | | | | | |
| 2 | -- | 8 | | | | | | | X | X | | | | | | |
| 5 | -- | 9 | | | | X | | | X | | X | X | | | | X |
| 3 | -- | 10 | | | | | | | | | X | X | X | | | |
| 3 | -- | 11 | | | | | | X | | | | X | X | | | |
| 3 | -- | 12 | | | | | | X | | | | | | X | X | |
| 4 | -- | 13 | | | | | | X | | | | | | X | X | X |
| 3 | -- | 14 | | | | | | | | | X | | | | X | X |



No. of fill-ins: EIGHT

Two fill-ins {(4,6), (6,4)}
Two fill-ins {(9,11), (11,9)}
Two fill-ins {(9,6), (6,9)}
Two fill-ins {(13,9), (9,13)}
No. of fill-ins: 8

(1) Row - 8 , column - 8 : Two non-zero elements → No fill-ins.

(2) Row - 1 , column - 1 : Three non-zero elements → No fill-ins.

(3) Row - 3 , column - 3 : Three non-zero elements → No fill-ins.

(4) Row - 10 , column - 10 : Three non-zero elements → Two fill-ins [ (9, 11) (11, 9) ].

(5) Row - 12 , column - 12 : Three non-zero elements → No fill-ins.

(6) Row - 14 , column - 14 : Three non-zero elements → Two fill-ins [ (13, 9) (9, 13) ].

(7) Row - 7 , column - 7 : Four non-zero elements → No fill-ins.

(8) Row - 11 , column - 11 : Four non-zero elements → Two fill-ins [ (9, 6) (6, 9) ].

(9) Row - 2 , column - 2 : Five non-zero elements → No fill-ins.

(10) Row - 13 , column - 13 : Five non-zero elements → No fill-ins.

(11) Row - 5 , column - 5 : Five non-zero elements → Two fill-ins [ (4, 6), (6, 4) ].

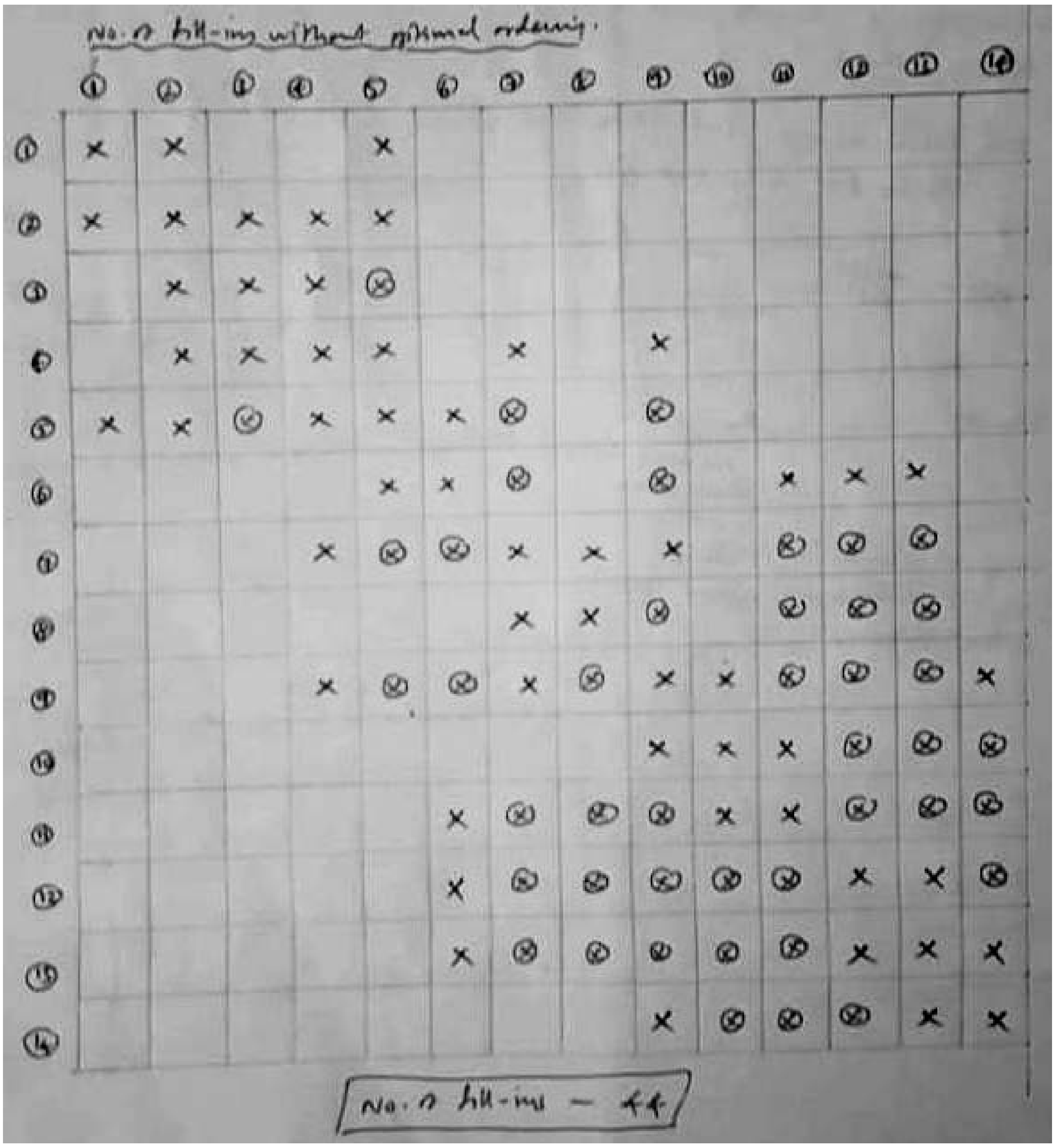
(12) Row - 4 , Column - 4 : Seven non-zero elements → No fill-ins.

(13) Row - 6 , column - 6 : seven non-zero elements → No fill-ins.

(14) Row - 9 , column - 9 : Eight non-zero elements → No fill-ins.

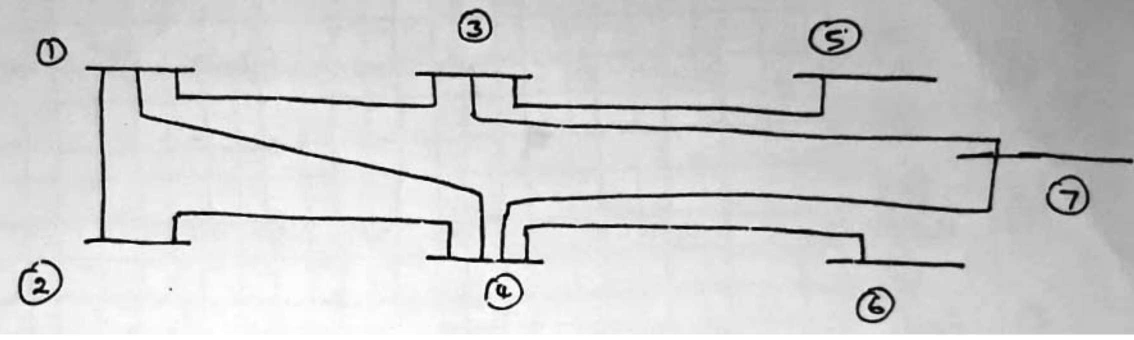
**OPTIMAL ORDER**: {8, 1, 3, 10, 12, 14, 7, 11, 2, 13, 5, 4, 6, 9}.

**PROBLEM:** **Compute the number of fill-ins in the above problem, if we do LU factorization without optimal ordering.**



EXAMPLE 2:

For a system shown in figure, using II scheme of optimal ordering, find the number of extra fill-ins that will exist during Triangular factorization before & after optimal ordering.

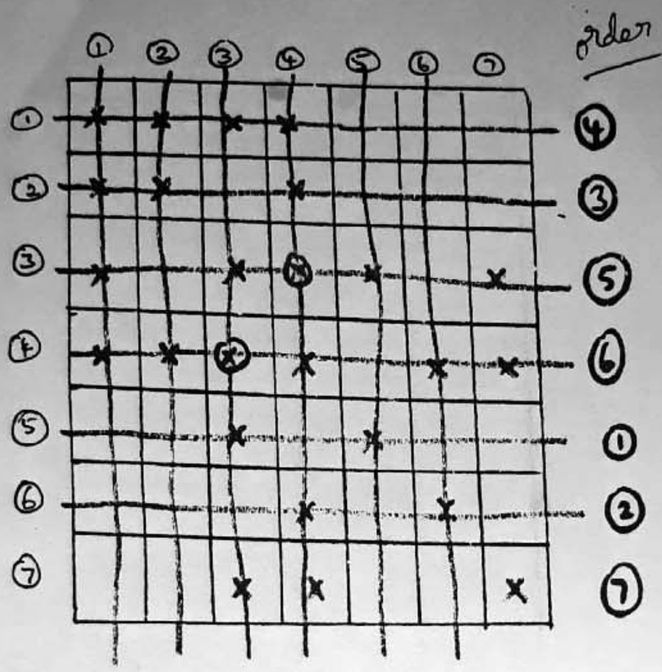



WITHOUT OPTIMAL ORDER:

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Non-Zeros | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | x | x | x | x | | | | 4 | fill ins: (2,3) (3,2) |
| 2 | x | x | ⊗ | x | | | | 3 | fill ins: (4,3) (3,4). |
| 3 | x | ⊗ | x | ⊗ | x | | x | 4 | fill ins: (5,4) (4,5) (7,5) (5,7) |
| 4 | x | x | ⊗ | x | ⊗ | x | x | 5 | fill ins: (5,6) (6,5) (7,6) (6,7) |
| 5 | | | x | ⊗ | x | ⊗ | ⊗ | 2 | No fill ins. |
| 6 | | | | x | ⊗ | x | ⊗ | 2 | No fill ins |
| 7 | | | x | x | ⊗ | ⊗ | x | 3 | No fill ins. |

No. of fill-ins : 12.

WITH OPTIMAL ORDERING:




optimal ordering = {5, 6, 2, 1, 3, 4, 7}

(1) Row - 5, column - 5 → Two non-zero elements → No fill-ins.

(2) Row - 6, Column - 6 → Two non-zero elements → NO fill-ins

(3) Row - 2, column - 2 → Three non-zero elements → NO fill-ins.

(4) Row - 1, Column - 1 → Four non-zero elements → 2 fill ins [(4,3), (3,4)]

(5) Row - 3, Column - 3 → Four non-zero elements → No fill ins

(6) Row - 4, Column - 4 → Five non-zero elements → No fill ins.

(7) Row - 7, column - 7 → NO fill ins.

# UNIT-III

## Load Flow Studies:-

Load flow or power flow analysis is a computer aided power system analysis to obtain the solution under static operating conditions. This analysis is carried out to determine

1. Bus voltages
2. Line flows
3. the effect of change in circuit configuration
4. the effect of loss of generation
5. economic system generation
6. transmission loss minimisation
7. possible improvement to an existing system by change of conductor size and system voltage.

For load flow analysis, a single phase representation of the power network is used since the system is generally balanced and the loads are represented by constant powers. In the network, at each bus, there are four variables viz.

1. Voltage magnitude
2. Voltage phase angle
3. Real power and
4. Reactive power.

| Bus | Specified variables | Computed variables |
|---|---|---|
| Slack bus | Voltage magnitude and its phase angle | Real and reactive powers |
| Generator bus (PV bus) | Magnitudes of bus voltages and real powers and limits on reactive powers | Voltage phase angle and reactive power |
| Load bus (PQ bus) | Real and reactive powers | Magnitude and phase angle of bus voltages |

Out of these four quantities, two of them are specified at each bus and the remaining two are determined from the load flow solution. To supply the real and reactive power losses in lines, which will not be known till the end of the power flow solution, a generator bus, called slack or swing bus is selected. At this bus, the generator voltage magnitude and its phase angle are specified so that the unknown power losses are also assigned to this bus in addition to balance of generation if any. Generally, at all other generator buses, voltage magnitude and real power are specified. At all load buses, the real and reactive load demands are specified. The following table illustrates the type of buses and the associated known and unknown variables.

At generator bus, through appropriate control of excitation and voltage regulating devices, it is possible to fix P and |V| and control Q to vary within certain limits with corresponding changes in δ. Besides controlling Q, it is possible to control the taps on the off-nominal transformers. With these control parameters, it is found that a larger set of feasible voltage profiles can be achieved. Thus it is clear that there is no unique load flow solution as such but a large number of alternative choices are possible for different sets of control parameters. A unique solution can be made by defining a cost or objective function such as minimising fuel cost or transmission losses or both. Such a formulation is called the optimal load flow problem

The day-to-day operational problems such as over-voltages, over frequency, over loads and so on should be solved very quickly by taking appropriate control action. such as reducing ~~the~~ the generation at some generator bus and increases the generation at some other generator bus, switching on shunt reactor or capacitor or adjusting the phase shifting transformer or shedding load at suitable buses. These decisions can not be taken by one's judgement but only on the basis of on-line power flow analysis. It is very clear that power flow analysis is an important analytical tool which helps the designer in designing the power system to meet the present and future demands and also helps in operating the P.S in an efficient manner.

The load flow problem of a P.S is described by a set of algebraic non-linear equations. These equations are solved by number of algorithms. Some of the generally used methods are

1. Gauss Seidal
2. Newton-Raphson
3. Decoupled N-R
4. Fast decoupled load flow etc.

The methods basically distinguish between themselves in the rate of convergence, storage requirement and time of computation.

## REPRESENTATION - POWER FLOW VARIABLES

Bus Voltage….

$$V_i = |V_i| \angle \delta_i = |V_i| \, e^{j\delta_i} = |V_i| \left(\cos\delta_i + j\sin\delta_i\right) = e_i + j\, f_i$$

Ybus element…..

$$Y_{ik} = |Y_{ik}| \angle \theta_{ik} = |Y_{ik}| \, e^{j\theta_{ik}} = G_{ik} + j\, B_{ik}$$

Bus Current….

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Bus Power….

$$S_i = P_i + j\,Q_i = V_i\, I_i^* = V_i \sum_{j=1}^{n} Y_{ij}^* V_j^*$$

Hybrid Form….

$$S_i = P_i + j\,Q_i = \sum_{j=1}^{n} |V_i V_j| \, e^{j(\delta_i - \delta_j)} \, (G_{ij} - jB_{ij})$$

Separating the real and imaginary parts …..

$$P_i = \sum_{j=1}^{n} |V_i V_j| \left\{ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right\}$$

$$Q_i = \sum_{j=1}^{n} |V_i V_j| \left\{ G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right\}$$

Polar Form……

$$S_i = P_i + j\,Q_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \, e^{j(\delta_i - \delta_j - \theta_{ij})}$$

Separating……

$$P_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$$

Rectangular Form……

$$S_i = P_i + j\,Q_i = \sum_{j=1}^{n} (e_i + j\,f_i)(G_{ij} - jB_{ij})(e_j - j\,f_j)$$

Separating……

$$P_i = \sum_{j=1}^{n} e_i(G_{ij}e_j - B_{ij}\,f_j) + f_i(G_{ij}\,f_j + B_{ij}e_j)$$

$$Q_i = \sum_{j=1}^{n} f_i(G_{ij}e_j - B_{ij}\,f_j) - e_i(G_{ij}\,f_j + B_{ij}e_j)$$

## POWER FLOW ANALYSIS

Power flow analysis is the determination of steady state conditions of a power system for a specified power generation and load demand. It basically involves the solution of a set of non-linear equations for the real and reactive powers at each bus.

It is used in the planning and design stages as well as during the operational stages of a power system. Certain applications, especially in the fields of power system optimization and distribution automation, require repeated fast power flow solutions. Due to a large number of interconnections and continuously increasing demand, the size and complexity of the present day power systems, have grown tremendously and it becomes very difficult to obtain power flow solutions, which is ideally suitable for real time applications. The three traditional methods used for power flow are

- Gauss Seidel (GS)
- Newton Raphson (NR)
- Decoupled NR
- FDLF

GS method was one of the most common method in power flow studies. This is the GS expression that may be solved iteratively for the solution of power flow problem. This method is simple, requires less computer memory but this method is slow due to poor rate of convergence, number of iterations increases directly with the system size and choice of slack bus affects the convergence of this algorithm. Because of these drawbacks, this method is not used for present day power systems.

NR method is very powerful technique in solving power flow problem. This is a gradient technique and needs the jacobian matrix to be formed during the iterative process. This Jacobian matrix provides the optimal direction for finding the solution. This method has several advantages. It reliably converges. It is insensitive to selection of slack bus. No of iterations is independent of system size. It requires less no of iterations. But it is very inefficient in the sense that it requires large computer memory and takes large computation time. That is why this algorithm is not suitable for real-time applications.

Simplifications in the jacobian tend to alter the direction, generally increasing the number of iterations. If the simplifications are done properly, an improvement in overall computational performance may be achieved. Whatever be the simplifications made, the final solution should remain unchanged.

There is weak coupling between Real power flow and Reactive power flow in power systems. Based on this weak coupling the real and reactive set of equations are decoupled and the problem is split into two sub-problems in FDLF. In this method, the jacobian matrices are made constant and need not be recomputed during the iterative process. It is developed with the following assumptions.

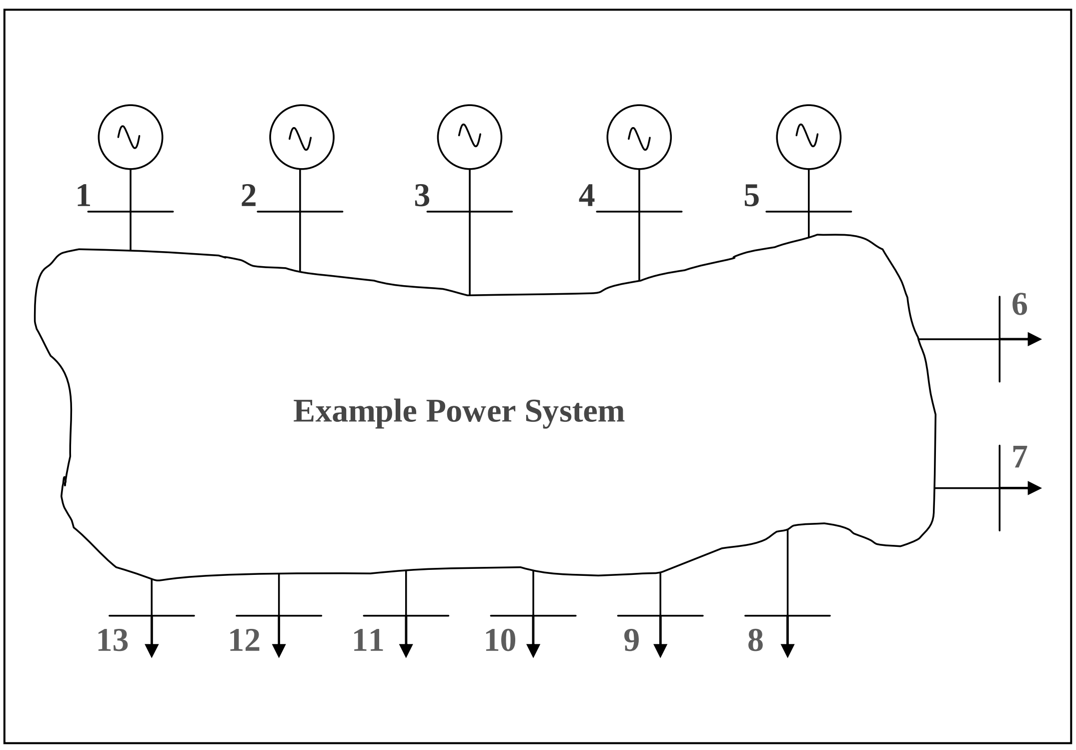
- the voltage magnitudes, $V$, are close to 1 p.u
- the phase angles, $\delta$, are not large in magnitude
- $r \ll x$.

This algorithm is fast and requires very less computer memory. This algorithm is predominantly used in the energy management systems, even for real time applications. However, it diverges, if any of the assumptions becomes invalid.

## Classification of Buses

| Bus | Specified | Computed |
|---|---|---|
| Slack | V, δ | P,Q |
| Generator ( PV ) | P, V | Q, δ |
| Load ( PQ ) | P, Q | V, δ |

## Example System with Known and Unknown variables




| | Slack bus | Generator Buses | | | | *Load Buses* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Specified | $V_1\delta_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | | | | | | | | |
| | | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $P_{13}$ |
| Unknown **12** | | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$ | $\delta_8$ | $\delta_9$ | $\delta_{10}$ | $\delta_{11}$ | $\delta_{12}$ | $\delta_{13}$ |
| | | | | | | | | | | | | | |
| Specified | | | | | | $Q_6$ | $Q_7$ | $Q_8$ | $Q_9$ | $Q_{10}$ | $Q_{11}$ | $Q_{12}$ | $Q_{13}$ |
| Unknown **8** | | | | | | $V_6$ | $V_7$ | $V_8$ | $V_9$ | $V_{10}$ | $V_{11}$ | $V_{12}$ | $V_{13}$ |

## Gauss-Seidel method :-

The Gauss Seidel method, which is used to solve a load flow problem, is an iterative algorithm for solving a set of non-linear algebraic equations.

The performance eqn of a power system can be written as

$$[I_{bus}] = [Y_{bus}][V_{bus}] \quad \sim ①$$

Selecting one of the busses as the reference bus (usually slack bus), we will get nb-1 simultaneous eqns.

The bus loading eqn can be written as

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad \sim ②$$

$i = 1, 2 \dots nb$

$i \neq$ slack bus.

From ①

$$I_i = \sum_{j=1}^{nb} Y_{ij} \cdot V_j \quad \sim ③$$

Equating ② and ③, we get

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{nb} Y_{ij} \cdot V_j$$

Rearranging the above

$$V_i = \frac{1}{Y_{ii}} \left( \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{nb} Y_{ij} \cdot V_j \right)$$

$i = 1, 2 \dots nb$

$i \neq$ slack bus.

If latest available voltage is used in RHS of the above eqn, we get

$$V_i^{new} = \frac{1}{Y_{ii}} \left( \frac{P_i - jQ_i}{V_i^{old\,*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{new} - \sum_{j=i+1}^{nb} Y_{ij} \cdot V_j^{old} \right) \quad ④$$

The above equation can be solved for bus voltages in an iterative manner. In a load flow problem, $P_s'$ at all buses except slack bus are specified. Illy $Q_s'$ at all load buses (PQ) are specified. For generator (PV) buses $Q_s'$ are not specified. Only its limits are specified. During the iterative process, Q for PV bus must be calculated using the following eqn. and must be substituted in the G.S. algorithm.

$$Q_i^{cal} = \text{Imag}\left(V_i \cdot I_i^*\right) = \text{Imag}\left(V_i \sum_{j=1}^{nb} Y_{ij}^* \cdot V_j^*\right)$$

Since the voltage at all generator buses must be maintained at $|V_i|^{sp}$, the real and imaginary parts of $V_i^{k+1}$ are adjusted as follows.

$$\delta_i^{k+1} = \tan^{-1} \frac{f_i^{k+1}}{e_i^{k+1}}$$

$$V_i^{k+1} = |V_i|^{sp} \cdot \cos\left(\delta_i^{k+1}\right) + j\,|V_i|^{sp} \cdot \sin\left(\delta_i^{k+1}\right)$$

The reactive power limit of all PV buses are taken into account by the following logic.

if $Q_i^{cal} > Q_i^{max}$, set $Q_i^{cal} = Q_i^{max}$.

if $Q_i^{cal} < Q_i^{min}$, set $Q_i^{cal} = Q_i^{min}$.

If any one of the above is satisfied, (ie limits are violated) for a PV bus, then that bus may be treated as a PQ bus and ~~not in~~ there is no need for voltage mag. adjustment. If in the subsequent computations, $Q_i^{cal}$ does fall within the available reactive power range, the bus is switched back to a P-V bus.

## Acceleration of Convergence :-

The G-S algorithm converges slowly because, in a large network, each bus may be connected to 3 or 4 other buses. This results in a "weak" mathematical coupling of the iterative scheme. So, Acceleration techniques are used to speed up the convergence. After every iteration, a correction is applied to each PQ bus voltage as follows.

$$\Delta V_i^{k+1} = \alpha . \left( V_i^{k+1} - V_i^{k} \right)$$

and new voltage will be

$$V_i^{(k+1)} = V_i^{k} + \Delta V_i^{k+1}$$

The acceleration factor 'α' in the above eqn is empirically determined between 1 and 2. ie $(1 < \alpha < 2)$.

## Newton - Raphson Method

The N-R method is a powerful method of solving a set of non-linear algebraic equations. It works faster and is sure to converge in most of the cases as compared to G-S method

Its only drawback is the large requirement of computer memory, which can be overcome by compact storage scheme.

In power flow problem, the complex bus voltages of the system are to be determined in such a way that the specified powers are satisfied. The real powers are specified at all buses except slack bus (n-1) and reactive powers are specified at all load buses (n-m) Therefore, the load flow problem is described by a set of algebraic non-linear equations as

$$\left.\begin{aligned} P(\delta, V) - P^{sp} &= 0 \\ Q(\delta, V) - Q^{sp} &= 0 \end{aligned}\right\} \quad (1)$$

where

$\delta$ = voltage angles at all buses except slack bus

$V$ = voltage magnitude at all load buses. The voltage magnitude at all PV buses are specified & known

$$P_i(\delta, V) = V_i \sum_j V_j \left[G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)\right]$$

$i = 2, 3 \ldots n$

$$Q_i(\delta, V) = V_i \sum_j V_j \left[G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)\right]$$

$i = m+1 \ldots n$

$P_i^{sp}$ = specified real power at bus - $i$

$Q_i^{sp}$ = specified reactive power at bus - $i$

It should be noted that while computing the above functions, the specified voltage magnitude of PV buses are to be substituted in place of the variable $V_i$ ; $i = 1, 2 .. m$ .

The above equations can be written in terms of correction variables $\Delta\delta$ and $\Delta V$ as

$$\left.\begin{aligned} P\left([\delta^0+\Delta\delta],[V^0+\Delta V]\right) - P^{sp} &= 0 \\ Q\left([\delta^0+\Delta\delta],[V^0+\Delta V]\right) - Q^{sp} &= 0 \end{aligned}\right\} \quad — (2)$$

where

$\delta^0$ and $V^0$ are the values of $\delta$ and $V$ corresponding to initial guess and $\Delta\delta$ & $\Delta V$ are the correction values such that the above equations are satisfied

The above equations can be expanded by Taylor's series as follows.

$$P(\delta^0,V^0) + \left.\frac{\partial P}{\partial \delta}\right|_{\substack{\delta=\delta^0 \\ V=V^0}} \Delta\delta + \left.\frac{\partial P}{\partial V}\right|_{\substack{\delta=\delta^0 \\ V=V^0}} \Delta V + \dots - P^{sp} = 0$$

$$Q(\delta^0,V^0) + \left.\frac{\partial Q}{\partial \delta}\right|_{\substack{\delta=\delta^0 \\ V=V^0}} \cdot \Delta\delta + \left.\frac{\partial Q}{\partial V}\right|_{\substack{V=V^0 \\ \delta=\delta^0}} \Delta V + \dots - Q^{sp} = 0$$

Neglecting the higher order derivatives, the above equations can be written in matrix form as

$$\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} - \begin{bmatrix} P^{sp} - P(\delta^0,V^0) \\ Q^{sp} - Q(\delta^0,V^0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad — (3)$$

Let

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P^{sp} - P^{cal} \\ Q^{sp} - Q^{cal} \end{bmatrix} = \begin{bmatrix} P^{sp} - P(\delta^0,V^0) \\ Q^{sp} - Q(\delta^0,V^0) \end{bmatrix}$$ is the mismatch vector

Eq (3) can then be written as

$$\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad 2\,(4)$$

Jacobian matrix     correction vector     Mismatch vector

In order to make the jacobian matrix symmetrical, the above eqn can be modified as

$$\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}|V| \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}|V| \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V/|V| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad 2\,(5)$$

$$\begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V/|V| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \qquad 2\,(6)$$

if $i \neq j$

$$H_{ij} = \frac{\partial P_i}{\partial \delta_j} \quad ; \quad N_{ij} = \frac{\partial P_i}{\partial V_j}|V_j|$$

$$M_{ij} = \frac{\partial Q_i}{\partial \delta_j} \quad ; \quad L_{ij} = \frac{\partial Q_i}{\partial V_j}|V_j|$$

$$2\,(7)$$

if $i = j$

$$H_{ii} = \frac{\partial P_i}{\partial \delta_i} \quad ; \quad N_{ii} = \frac{\partial P_i}{\partial V_i}|V_i|$$

$$M_{ii} = \frac{\partial Q_i}{\partial \delta_i} \quad ; \quad L_{ii} = \frac{\partial Q_i}{\partial V_i}|V_i|$$

Eq. (6) may be solved iteratively to obtain the load flow solution. Convergence check is carried out using $\Delta P$ and $\Delta Q$ vectors.

During the iterative process, if any of the reactive power generation at PV buses violates the reactive power limit, then the reactive power generation at that bus is set to the respective limit and then that particular bus is treated as a load bus in the subsequent iterations. This obviously alters the jacobian matrix and the corresponding mismatch and correction vectors as

$$\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V}|V| & \frac{\partial P}{\partial V'}|V'| \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}|V| & \frac{\partial Q}{\partial V'}|V'| \\ \frac{\partial Q'}{\partial \delta} & \frac{\partial Q'}{\partial V}|V| & \frac{\partial Q'}{\partial V'}|V'| \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/|V| \\ \Delta V/|V'| \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \\ \Delta Q' \end{bmatrix}$$

where

$\Delta Q'$ = mismatch reactive power vector of limit violated generators

$\Delta V'$ = corrections of voltage magnitude of limit violated generators

3rd row and 3rd col of jacobian matrix represent the additional derivatives corresponding to the limit violated generators

## Algorithm

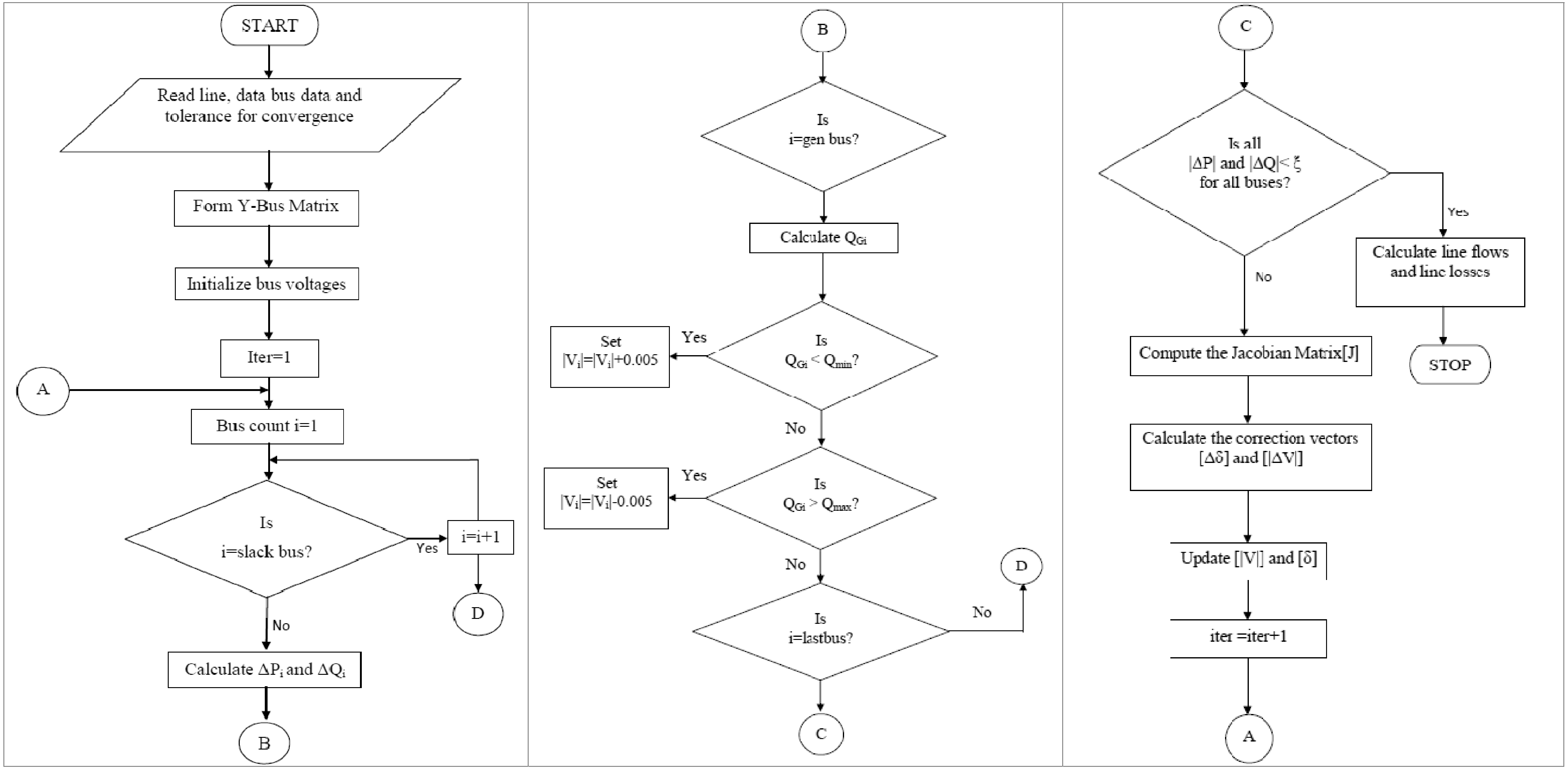
1. Form Ybus matrix
2. Initialize all bus voltage magnitudes and angles
3. Calculate mismatch real powers $[\Delta P]$ at all buses except slack bus
4. Calculate the reactive power generations at all PV buses and check for Q-limit violations. If any of the generator exceeds the limit, set the value to its respective limit and treat this as a PQ bus
5. Calculate mismatch reactive powers $\begin{bmatrix}\Delta Q \\ \Delta Q'\end{bmatrix}$ at all load buses and limit violated PV buses.
6. Check for convergence. i.e., check whether all the elements in $[\Delta P]$ and $\begin{bmatrix}\Delta Q \\ \Delta Q'\end{bmatrix}$ are within a specified tolerance value. If converged, goto step 10
7. Form the jacobian matrix taking into account the generator reactive power limit violations.
8. Solve Eq. 8 for $\begin{bmatrix}\Delta \delta \\ \Delta V/|V| \\ \Delta V'/|V'|\end{bmatrix}$ and update the vectors.

$$V_i^{new} = V_i^{old} + \frac{\Delta V_i}{|V_i|} * V_i^{old}$$

$$\delta_i^{new} = \delta_i^{old} + \Delta \delta_i$$

9. Goto step 3
10. ~~Calculate~~ Calculate all line flows, slack bus power and reactive power generations at all generator buses and print the results.

## Flow Chart of NR Method



## Decoupled N-R method :-

In any practical power systems, the changes in real power is more dependent on the changes in voltage angles at various buses than the changes in voltage magnitudes; and the changes in reactive power at a bus is more dependent on the changes in voltage magnitudes at various buses than the changes in voltage angles. Thus, there is a fairly good decoupling betw. the active power and reactive power. This decoupling feature can be used in simplifying the N-R algorithm by neglecting [N] and [M] in the jacobian matrix.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V/|V| \end{bmatrix}$$

$$[\Delta \delta] = [H]^{-1} [\Delta P] \quad \text{—③}$$

$$\left[\frac{\Delta V}{|V|}\right] = [L]^{-1} [\Delta Q] \quad \text{—④}$$

Eqns. ③ & ④ can be solved simultaneously at each iteration. A better approach is to first solve eqn ③ for $\Delta\delta$ and use the updated $\delta$ to construct and solve eqn ④. for $\Delta V$. This will result in faster convergence than the simultaneous mode.

**Advantages**

1. Memory requirement is reduced compared to formal N-R method

2. Though the number of iterations increase, the overall computation is reduced than the formal N-R method.

## Fast Decoupled Load Flow Method :-

The FDLF method is very fast method of obtaining load flow solution. In this method, both the speed as well as the sparsity are exploited. This is actually an extension of N-R method.

The N-R method is

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V/|V| \end{bmatrix}$$

The decoupled N-R method is

$$[\Delta P] = [H][\Delta\delta] \qquad (5)$$

$$[\Delta Q] = [L][\Delta V/|V|]$$

based on decoupling between real and reactive powers. This decoupled N-R method is further simplified using the following assumptions.

$$\cos(\delta_p - \delta_q) \simeq 1.0$$

The system is usually not designed to operate at its max. steady state stability limit. So, the voltage angle difference betw. the terminal buses of any trans. line is less than 30°. So, the cosine of this angle diff. is approximately one.

$$G_{pq} \cdot \sin(\delta_p - \delta_q) << B_{pq}$$

In any H.V. trans. system, $R_{pq} << X_{pq}$. In addition $\sin(\delta_p - \delta_q) = (\delta_p - \delta_q)$, which is a smaller value. So, the product $G_{pq} \cdot \sin(\delta_p - \delta_q)$ is less than $B_{pq}$.

$$Q_p << B_{pp} |V_p|^2$$

With these assumptions, the Jacobian terms can be written as

$p \neq q$

$$H_{pq} = L_{pq} = -|V_p|\,|V_q|\,B_{pq}$$

$p = q$

$$H_{pp} = L_{pp} = -B_{pp} |V_p|^2.$$

With these assumptions, eqn ⑤ can be written as

$$[\Delta P] = \left[ |V_p| \, |V_q| \, B'_{pq} \right] [\Delta \delta]$$

$$[\Delta Q] = \left[ |V_p| \, |V_q| \, B''_{pq} \right] \left[ \Delta V / |V| \right] \quad \text{— ⑥}$$

where $B'_{pq}$ and $B''_{pq}$ are the elements of $[-B]$ matrix

The final FDLF algorithm can be achieved by the following simplifications.

1. neglecting the elements that predominentaly affect reactive power flows, such as shunt reactances, tap changing transformer etc, while forming $B'$ matrix

2. neglecting the elements that predominentaly affect real power flows such as phase shifting transformer, while forming $B''$ matrix.

3. neglecting the series resistance in calculating the elements of $B'$ matrix.

4. dividing each of the eqn ⑥ by $[V_p]$ and setting $V_q = 1$ pu.

With these assumptions, the final FDLF eqn becomes

$$\left[ \frac{\Delta P}{|V|} \right] = [B'] [\Delta \delta] \quad \text{— ⑦}$$

$$\left[ \frac{\Delta Q}{|V|} \right] = [B''] [\Delta V] \quad \text{— ⑧}$$

- both $B'$ & $B''$ are real and sparse and have structures of $[H]$ and $[L]$ respectively.

- since they contain network admittance, they are constant and need to be evaluated only once at the beginning of the study.

- both $B'$ & $B''$ are symmetric, if phase shifting transformers are not present.

The equations ⑦ & ⑧ are solved alternatively always employing the most recent voltage values. i.e. the eqn ⑦ is solved for $[\Delta\delta]$ and the updated value of $[\delta]$ is used to solve eqn ⑧ for $[\Delta V]$. Separate convergence tests are applied for the real and reactive power mismatches.

Q-limit violations :-

- Q limit violation may be taken into account similar to formal N-R method.

- if reactive power at any generator bus violates, the violated bus is treated as PQ bus by setting the reactive generation to its respective limit and the bus is treated as load bus; and ~~alter~~ alter the $B''$ matrix accordingly.

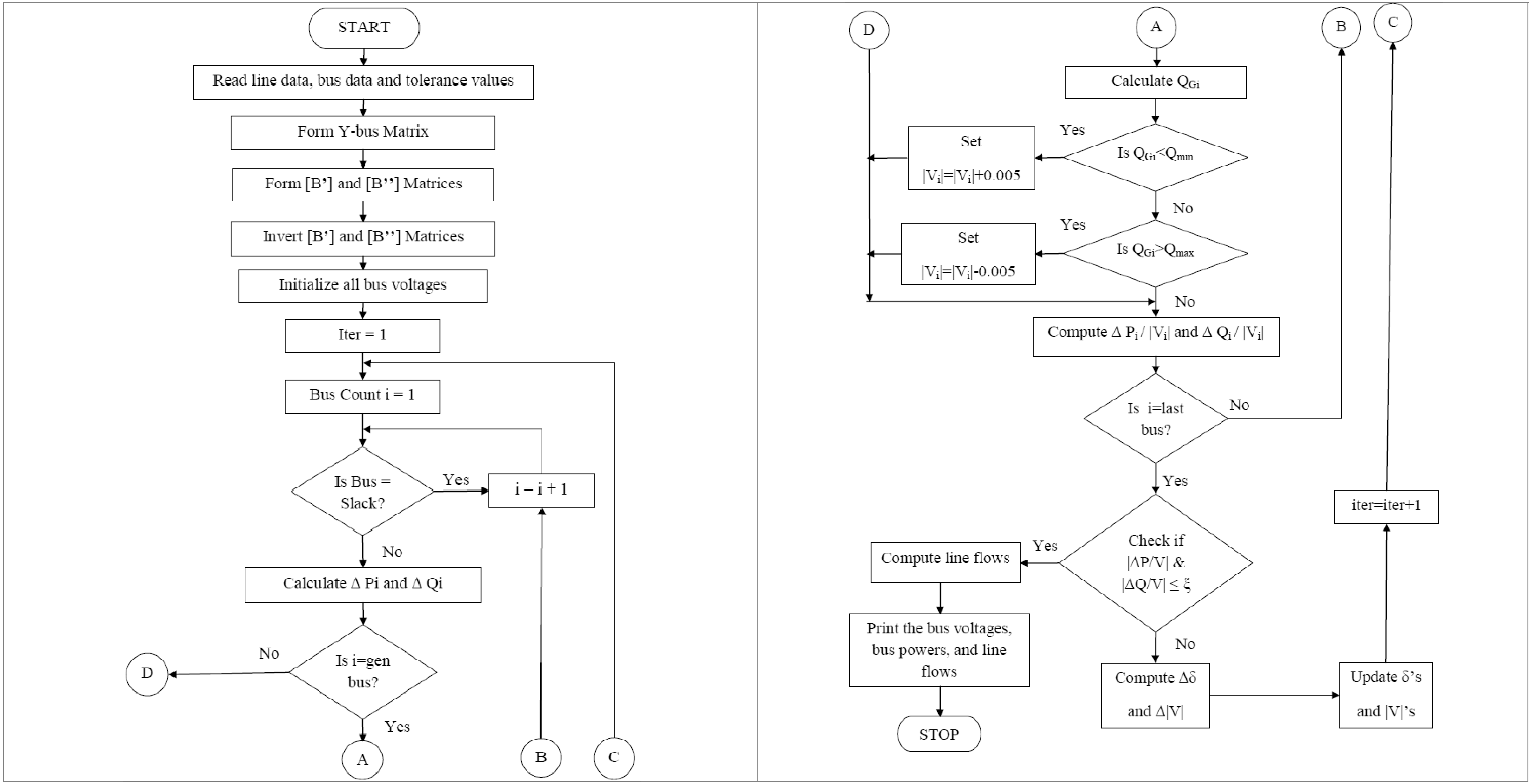
Features :-

- it takes more no. of iterations.
- it is more reliable
- it requires less computer memory
- it is faster

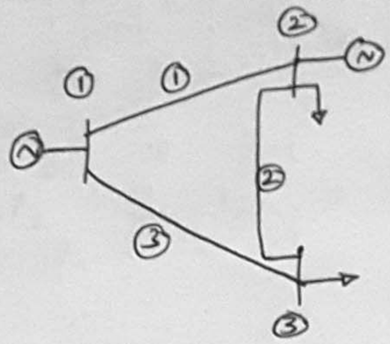
than the formal N-R method

- it fails if $X \gg R$ of trans. lines are not satisfied.

## Flow Chart of FDLF Method



For the system shown below, carry out one iteration of the FDLF and hence find out voltages and angles at all the buses.



Line data:

| Line No | Buses | Line Impedance | Half-line charging Admittance |
|---|---|---|---|
| 1 | 1 – 2 | j0.1 | j0.01 |
| 2 | 2 – 3 | j0.2 | j0.015 |
| 3 | 1 – 3 | j0.2 | j0.015 |

Bus Data: (all quantities in per unit)

| Bus No. | Type | Generation | | Load | | \|V\| | Q limits | |
|---|---|---|---|---|---|---|---|---|
| | | P | Q | P | Q | | Q min | Q max |
| 1 | Slack | — | — | — | — | 1.0 | — | — |
| 2 | PV | 5.32 | — | 0.8 | 0.1 | 1.05 | 0 | 6.0 |
| 3 | PQ | — | — | 3.64 | 0.54 | — | — | — |

Soln

Step: 1

$$Y'_{bus} \text{ (w/o line charging admittance)} = \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -j15 & j5 \\ j5 & j5 & -j10 \end{bmatrix} ; \quad B' = \begin{bmatrix} 15 & -5 \\ 5 & -10 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} 0.08 & 0.04 \\ 0.04 & 0.12 \end{bmatrix}$$

$$Y''_{bus} \text{ (with line charging admittance)} = \begin{bmatrix} -j14.975 & j10 & j5 \\ j10 & -j14.975 & j5 \\ j5 & j5 & -j9.97 \end{bmatrix} ; \quad B'' = \begin{bmatrix} 9.97 \end{bmatrix}$$

$$[B'']^{-1} = \begin{bmatrix} 0.1 \end{bmatrix}$$

Step: 2

$$[V] = \begin{bmatrix} 1+j0 \\ 1.05+j0 \\ 1+j0 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 1.05\angle 0^\circ \\ 1\angle 0^\circ \end{bmatrix}$$

Step: 3

$$\left[\frac{\Delta P}{|V|}\right] = \begin{bmatrix} \Delta P_2 / V_2 \\ \Delta P_3 / V_3 \end{bmatrix}$$

$$\Delta P_2 = P_2^{sp} - P_2^{cal} = [5.32 - 0.8] - 0 = 4.52$$

where

$$P_2^{cal} = V_2 V_1 Y_{21} \cos(\delta_2 - \delta_1 - 90) + V_2^2 Y_{22} \cos(\delta_2 - \delta_2 - 90^\circ) + V_2 V_3 Y_{23} \cos(+\delta_2 - \delta_3 - 90^\circ)$$
$$= 0.0$$

$$\frac{\Delta P_2}{V_2} = \frac{4.52}{1.05} = 4.3.$$

$$\Delta P_3 = P_3^{sp} - P_3^{cal} = -3.64 - 0 = -3.64$$

$$\frac{\Delta P_3}{V_3} = \frac{-3.64}{1} = -3.64$$

where

$$P_3^{cal} = V_3 V_1 Y_{31} \cos(\delta_3 - \delta_1 - \theta_{31}) + V_3 V_2 Y_{32} \cos(\delta_3 - \delta_2 - \theta_{32}) + V_3^2 Y_{33} \cos(\delta_3 - \delta_3 - \theta_{33})$$
$$= 0 \quad \Big\| \text{ as all } \delta's = 0 \text{ and } \theta's = 90^\circ$$

Step: 4

$$\left[\frac{\Delta Q}{|V|}\right] = \left[\frac{\Delta Q_3}{V_3}\right]$$

$$\Delta Q_3 = Q_3^{sp} - Q_3^{cal} = -0.54 - (-0.28) = -0.26$$

$$\frac{\Delta Q_3}{V_3} = \frac{-0.26}{1} = -0.26$$

where

$$Q_3^{cal} = V_3 V_1 Y_{31} \sin(\delta_3 - \delta_1 - \theta_{31}) + V_3 V_2 Y_{32} \sin(\delta_3 - \delta_2 - \theta_{32}) + V_3^2 Y_{33} \sin(\delta_3 - \delta_3 - \theta_{33})$$
$$= 1 \times 1 \times 5 \times \sin(-90) + 1 \times 1.05 \times 5 \times \sin(-90) + 1^2 \times 9.97 \times \sin(+90) = -0.28$$

Step: 5 Q-limit violation

$$Q_{min} \le Q_G \le Q_{max} \quad ; \quad 0 \le Q_G^2 \le Q_{max}.$$

$$Q_2^{cal} = V_2 V_1 Y_{21} \sin(\delta_2 - \delta_1 - \theta_{21}) + V_2^2 Y_{22} \sin(-\theta_{22})$$
$$+ V_2 V_3 Y_{23} \sin(\delta_2 - \delta_3 - \theta_{23})$$
$$= 1.05 \times 1 \times 10 \times \sin(-90) + 1.05^2 \times 14.975 \times \sin(+90^\circ)$$
$$+ 1.05 \times 1 \times \sin(-90) = -10.5 + 16.51 - 5.25$$
$$= 0.76$$

$$Q_G^2 = \underset{\substack{\uparrow \\ \text{local load.}}}{0.76 + 0.1} = 0.86 \text{ pu.}$$

$Q_G^2$ is with in the limits. So, there is no need to modify $B''$ and $\left[\frac{\Delta Q}{V}\right]$ ~~vector~~.

Step: 6 Convergence check.

Not converged as the values in $\frac{\Delta P}{V}$ & $\frac{\Delta Q}{V}$ are not small

Step: 7 Compute $\Delta\delta$ & $\Delta V$

$$\Delta\delta = \begin{bmatrix} 0.08 & 0.04 \\ 0.04 & 0.12 \end{bmatrix} \begin{bmatrix} 4.3 \\ -3.64 \end{bmatrix} = \begin{bmatrix} 0.1984 \\ -0.2648 \end{bmatrix}$$

$$\Delta V = [0.1]\,[-0.26] = -0.026$$

Step: 8 Update V & $\delta$

$$[\delta] = \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1984 \\ -0.2648 \end{bmatrix} = \begin{bmatrix} 0.1984 \\ -0.2648 \end{bmatrix}$$

$$[V] = [V_3] = [1] + [-0.026] = 0.974.$$

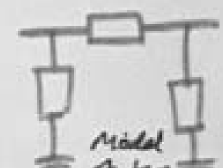
## UNIT-IV SHORT CIRCUIT STUDIES

### Short Circuit Analysis

Assumptions:- In short circuit studies, a number of assumptions are made to reduce the complexity of the problem. In general sufficient accuracy in the results is obtained with these assumptions. The various assumptions are as follows.

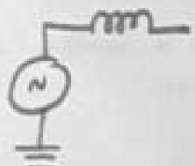
① During fault, the bus voltages drop very low and the currents drawn by the loads can be neglected in comparison to fault currents. So all loads, line charging capacitances, and other shunt connections to the ground are neglected.

② All tap changing transformers are assumed to be set at their nominal taps. This vanishes the shunt connection of the transformer and only the series reactance is considered.



Model of tap changing transformer.

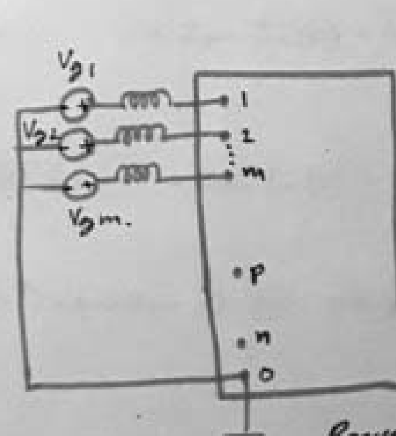
③ The generator is represented by a voltage source in series with a reactance which is taken as the subtransient or transient reactance.




④ If the resistances of the transmission lines are smaller than the reactances by a factor of six or more, the resistances are neglected. For high voltage systems $X/R \geq 6$ and hence R is neglected.

Symmetrical short circuit analysis :-

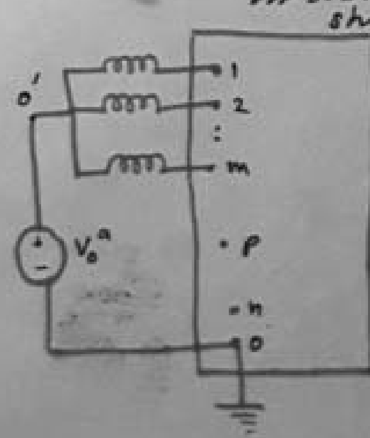
Let the transmission network consists of 'n' buses excluding the ground bus, denoted by 0. The first 'm' buses are assumed as the generator buses. By assumption, all these generator voltages are assumed to be equal. So, all the generators are augmented by a single generator connected betw. the fictitious node 0' and ground 0. as shown in fig. 2.




Power system representation for short ckt studies.

First consider $V_0^a$ is shorted. Then for the resulting passive network, Zbus can be obtained as

$$V_{bus} = Z_{bus} \cdot I_{bus} \quad \text{—①}$$




Now introduce the voltage source $V_0^a$ betw. 0' and 0. Now, modified n-port description is obtained by adding $V_0^a$ to all the equations.

$$V_{bus} = Z_{bus} \cdot I_{bus} + b \cdot V_0^a \qquad (2)$$

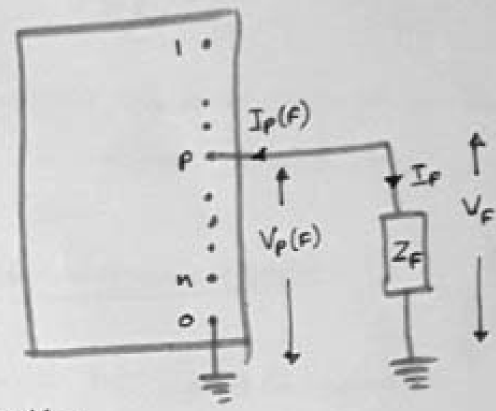
where $b = [1, 1, \ldots \ldots 1]^{tr}$

case : i

**3ϕ to ground fault – fault in impedance form :-**

Suppose the fault occurs at the $p^{th}$ bus. Let the fault impedance be $Z_F$ and the faulted network is described by

$$V_F = Z_F \cdot I_F \qquad (3)$$




The constraints betw. the variables of the fault impedance and port variables are written by inspection as

$$I_F = -I_p(F) \;;\; V_F = V_p(F) \;;\; I_i(F) = 0 \qquad i = 1, 2 \ldots n \;;\; i \neq p \qquad (4)$$

where $V_p(F)$ and $I_p(F)$ are the $p^{th}$ bus voltage & current respectively.

By expanding eqn (2), we get

$$\begin{aligned}
V_1(F) &= Z_{11} I_1(F) + \cdots + Z_{1p} I_p(F) + \cdots + Z_{1n} I_n(F) + V_0^a \\
V_2(F) &= Z_{21} I_1(F) + \cdots + Z_{2p} I_p(F) + \cdots + Z_{2n} I_n(F) + V_0^a \\
&\vdots \\
V_p(F) &= Z_{p1} \cdot I_1(F) + \cdots + Z_{pp} \cdot I_p(F) + \cdots + Z_{pn} \cdot I_n(F) + V_0^a \\
&\vdots \\
V_n(F) &= Z_{n1} \cdot I_1(F) + \cdots + Z_{np} \cdot I_p(F) + \cdots + Z_{nn} \cdot I_n(F) + V_0^a .
\end{aligned} \qquad (5)$$

substituting the relations (3) & (4) in the $p^{th}$ equation of (5), we get

$$Z_F \cdot I_F = -Z_{pp} \cdot I_F + V_0^a$$

$$\boxed{I_F = \frac{V_0^a}{Z_{pp} + Z_F}} \qquad (6)$$

$$V_F = V_p(F) = Z_F \cdot I_F = Z_F \cdot \frac{V_0^a}{Z_{pp} + Z_F} . \qquad (7)$$

The other bus voltages are obtained from the rest of the eqns in eqn. 5

$$\boxed{V_i(F) = V_0^a - Z_{ip} \cdot I_F} \qquad i = 1, 2, \ldots n \quad i \neq p. \qquad \text{— (8)}$$

This determines all bus voltages in the system, which in turn will determine the line currents in all the lines by elementary application of ohm's law.

case ii

## 3φ to ground fault – fault in admittance form

Let the fault admittance be $Y_F$ and the faulted network is described by

$$I_F = Y_F \cdot V_F. \qquad \text{— (9)}$$

Substituting 9 & 4 in the $p^{th}$ equation of 5, we get

$$V_F = V_p(F) = Z_{pp} \cdot I_p(F) + V_0^a$$

$$V_F = Z_{pp}\left(-Y_F \cdot V_F\right) + V_0^a$$

$$\boxed{V_F = \frac{V_0^a}{1 + Z_{pp} \cdot Y_F}} \qquad \text{— (10)}$$

$$\boxed{I_F = Y_F \cdot V_F.} \qquad \text{— (11)}$$

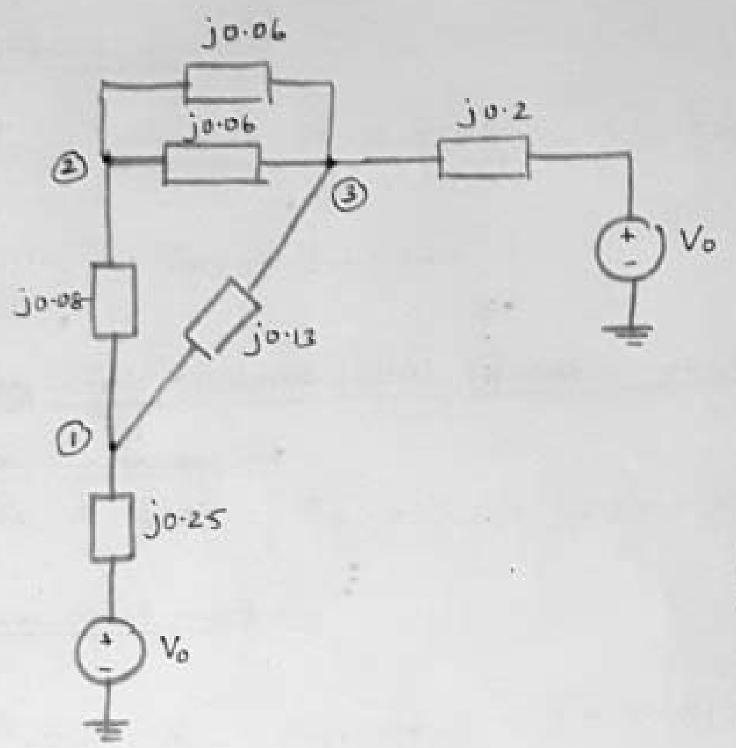
The other bus voltages can be obtained using eqn 8.

case iii

## 3φ symmetrical fault – not involving ground.

Since there is no impedance description for this fault, we can represent the fault by (+)ve sequence admittance and eqns 9, 10 and 11 can be used to carry out the fault analysis.

Problem
M.A.PAI
pp. 87




The positive sequence network of a three-bus power system is shown in fig. For a symmetrical 3ϕ to ground fault with $Z_f = j0.1$ p.u. Find the fault currents for faults at buses 1, 2 and 3. For fault at bus 1, find all bus voltages and line currents. Assume $V_0^a = 1\angle 0$.

Soln

While forming the Ybus matrix and also in all the calculations, we need not ~~put~~ put 'j'. Maybe in the final results, we can put 'j' appropriately.

$$
[Y_{bus}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c}
\begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{bmatrix} {}_{A}\,24.1923 & {}_{B}\,-12.5 & {}_{C}\,-7.6923 \\ {}_{D}\,-12.5 & {}_{E}\,45.83 & {}_{F}\,-33.33 \\ \underset{X}{-7.6923} & \underset{Y}{-33.33} & \underset{M}{46.0223} \end{bmatrix}
\end{array}
$$

$$
\text{cofactor} = \begin{bmatrix} \overset{EMYF}{+(498.3131)} & \overset{DMXF}{-(-831.663)} & \overset{DYXE}{+(769.1631)} \\ \overset{BMYC}{-(-831.663)} & \overset{AMXC}{+(1054.216)} & \overset{AYXB}{-(-902.48)} \\ \overset{BFEC}{+(769.163)} & \overset{AFDC}{-(-902.48)} & \overset{AEDB}{+(952.483)} \end{bmatrix} \qquad |\Delta| = 7839.1667
$$

$$
[Z] = \begin{bmatrix} 0.1274 & 0.1061 & 0.0981 \\ 0.1061 & 0.1345 & 0.1151 \\ 0.0981 & 0.1151 & 0.1215 \end{bmatrix}
$$

Fault at bus 1

$$
I_{f_1} = \frac{V_0^a}{Z_{pp} + Z_f} = \frac{1}{0.1274 + 0.1} = 4.3995 = -j4.3995
$$

$$
I_1(f) = j4.3995
$$

Fault at bus 2

$$
I_{f_2} = \frac{1}{0.1345 + 0.1} = 4.2644 = -j4.2644
$$

$$
I_2(f) = j4.2644.
$$

Fault at bus 3 :

$$I_{F3} = \frac{1}{0.1215 + 0.1} = 4.5147 = -j4.5147$$

$$I_3(F) = -I_{F3} = j4.5147$$

Voltages at all buses, when fault is at bus 1

voltage at faulted bus

$$V_{F1} = V_1(F) = I_{F1} * Z_F = 4.3995 \times 0.1 = 0.44 \text{ p.u.}$$

voltage at all other buses

$$V_2(F) = V_0^a - Z_{21} \cdot I_{F1} = 1 - 0.1061 * 4.3995 = 0.5334$$

$$V_3(F) = V_0^a - Z_{31} \cdot I_{F1} = 1 - 0.0981 * 4.3995 = 0.5686.$$

Currents through the transmission lines :-

$$I_{ij} = \frac{V_i - V_j}{x_{ij}}$$

$$I_{12} = \frac{0.44 - 0.5334}{0.08} = -1.1675 = j1.1675$$

$$I_{13} = \frac{0.44 - 0.5686}{0.13} = -0.9892 = j0.9892$$

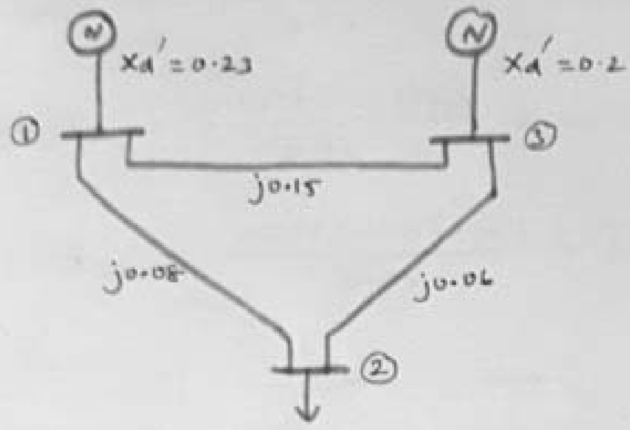
$$I_{23}(\text{line-1}) = I_{23}(\text{line-2}) = \frac{0.5334 - 0.5686}{0.06} = -0.5867$$
$$= j0.5867.$$

Generator currents :-

$$I_{G1} = \frac{V_0^a - V_1}{x_{g1}} = \frac{1 - 0.44}{0.25} = 2.24 = -j2.24 \text{ p.u}$$

$$I_{G3} = \frac{1 - 0.5686}{0.2} = 2.1570 = -j2.1570 \text{ p.u}$$

Problem. For the system shown in fig, calculate the fault current, bus voltages and generator currents when a 3$\phi$ to ground fault with $Z_F = j0.1$ p.u occurs at bus (2)




Soln

$$[Y_{bus}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \underset{A}{23.5145} & \underset{B}{-12.50} & \underset{C}{-6.6667} \\ \underset{D}{-12.50} & \underset{E}{29.1667} & \underset{F}{-16.6667} \\ \underset{X}{-6.6667} & \underset{Y}{-16.6667} & \underset{M}{28.3334} \end{bmatrix} \end{array} \quad \text{cofactor} = \begin{bmatrix} EMYF & DMXF & DYXE \\ BMYC & AMXC & AYXB \\ BFEC & AFDC & AEDB \end{bmatrix}$$

$$\text{cofactor} = \begin{bmatrix} 548.6129 & 465.2794 & 402.7794 \\ & 621.8008 & 475.2629 \\ & & 529.5904 \end{bmatrix} \qquad |\Delta| = 4398.9436$$

$$[Z_{bus}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.1247 & 0.1058 & 0.0916 \\ 0.1058 & 0.1414 & 0.1080 \\ 0.0916 & 0.1080 & 0.1204 \end{bmatrix} \end{array}$$

Fault at bus 2 with $Z_F = j0.1$

$$I_{F2} = \frac{1}{0.1414 + 0.1} = 4.1425 = -j4.1425$$

$$V_2 = I_{F2} * Z_F = 4.1425 * 0.1 = 0.4143$$

$$V_1 = 1 - 0.1058 * 4.1425 = 0.5617$$

$$V_3 = 1 - 0.1080 * 4.1425 = 0.5526$$

Generator currents

$$I_{G1} = \frac{1 - 0.5617}{0.23} = 1.9057 = -j1.9057$$

$$I_{G3} = \frac{1 - 0.5526}{0.2} = 2.237 = -j2.237$$

Transmission line flows :-

$$I_{12} = \frac{0.5617 - 0.4143}{0.08} = 1.8425 = -j1.8425$$

$$I_{13} = \frac{0.5617 - 0.5526}{0.15} = 0.0607 = -j0.0607$$

$$I_{32} = \frac{0.5526 - 0.4143}{0.06} = 2.3050 = -j2.3050.$$

— . — . —

Unsymmetrical Fault Analysis using Symmetrical components :-

Consider a general power network shown in fig 1. It is assumed that a shunt type of fault occurs at point p in the system, as a result of which currents $I_p^a, I_p^b, I_p^c$ flow out of the system and $V_p^a, V_p^b, V_p^c$ are voltages of line a, b, c with respect to ground.

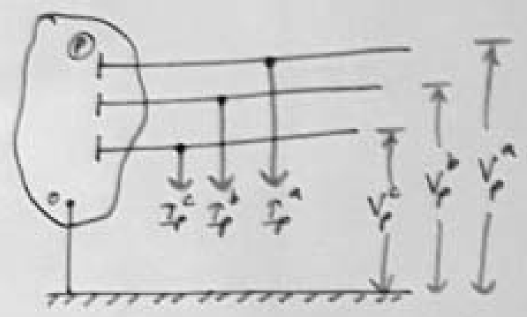


Fig:1 A general power system

At bus p, the prefault sequence voltage $\begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}^{012}$ is the open circuited thevenin's voltage and the sequence impedance viewed at point 'p' $\begin{bmatrix} Z_{pp}^0 & & \\ & Z_{pp}^1 & \\ & & Z_{pp}^2 \end{bmatrix}$ is the thevenin's impedance. Then, the thevenin's equivalent ckt at fault point 'p' is represented in ~~fit~~ fig. 2. From these networks, the voltage at point 'p' can be written as

$$V_p^0 = 0 - Z_{pp}^0 \cdot I_p^0$$

$$V_p^1 = \sqrt{3} - Z_{pp}^1 \cdot I_p^1$$

$$V_p^2 = 0 - Z_{pp}^2 \cdot I_p^2$$

It can be written in matrix form as

$$[V_p^{012}] = \begin{bmatrix} V_p^0 \\ V_p^1 \\ V_p^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{pp}^0 & & \\ & Z_{pp}^1 & \\ & & Z_{pp}^2 \end{bmatrix} \begin{bmatrix} I_p^0 \\ I_p^1 \\ I_p^2 \end{bmatrix} \quad 20$$

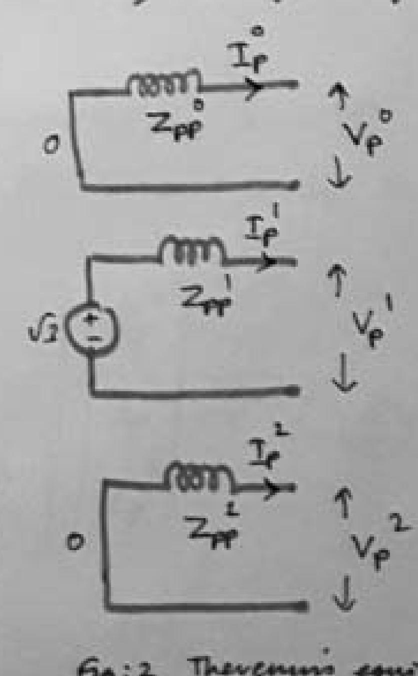


Fig:2 Thevenin's equivalent circuits as seen at fault point 'p'.

In the above eqn, the unknowns are $V_p^{012}$ and $I_p^{012}$. Depending upon the type of fault, the sequence network may be appropriately connected and the unknown parameters can then be easily computed. The various types of unsymmetrical faults are

1. Single line to ground fault (SLG)
2. Line to Line fault (LL)
3. Double line to ground fault. (LLG)

SLG fault :-

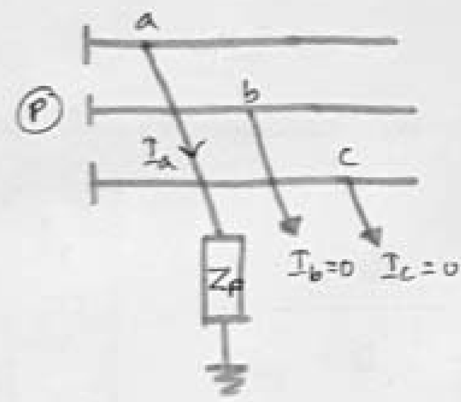



Fig: 3 SLG fault

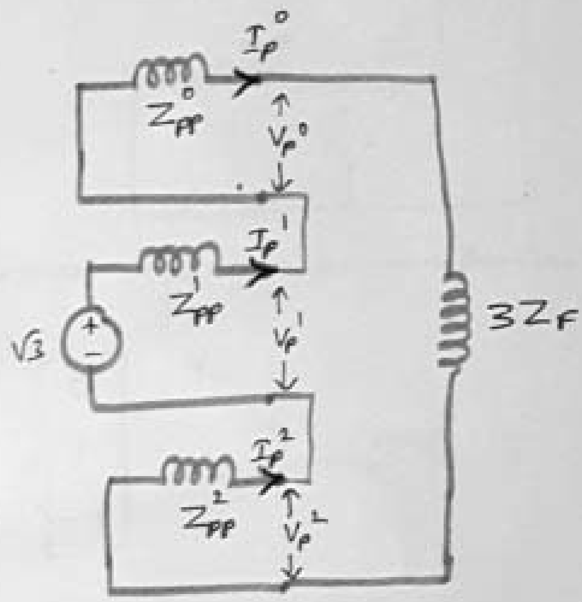



Fig: 4. Connection of Sequence networks for SLG fault.

Let the fault impedance be $Z_F$. Then the sequence network can be connected as shown in fig. 4. The fault current at faulted bus 'p' can be written as

$$\left[I_p^{012}\right] = \begin{bmatrix} I_p^0 \\ I_p^1 \\ I_p^2 \end{bmatrix} = \frac{\sqrt{3}}{Z_{pp}^0 + Z_{pp}^1 + Z_{pp}^2 + 3 \cdot Z_F} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{—} (2)$$

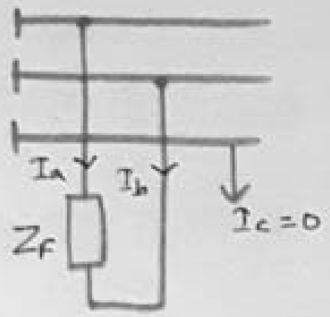
The fault bus voltages can be computed by eqn (1)

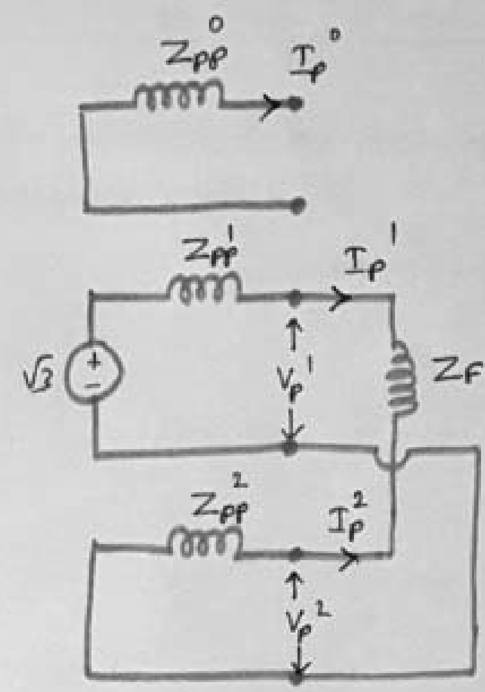
The voltages of all other buses can be computed by the following eqn:-

$$\left[V_q^{012}\right] = \begin{bmatrix} V_q^0 \\ V_q^1 \\ V_q^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{pq}^0 & & \\ & Z_{pq}^1 & \\ & & Z_{pq}^2 \end{bmatrix} \begin{bmatrix} I_p^0 \\ I_p^1 \\ I_p^2 \end{bmatrix} \quad \text{—} (3)$$

$q = 1, 2 \ldots nb$
$q \neq p$

## LL fault :-




$$I_p^0 = 0$$

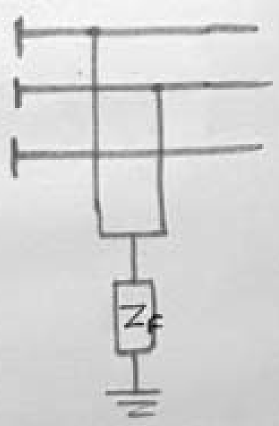
$$I_p^1 = -I_p^2$$

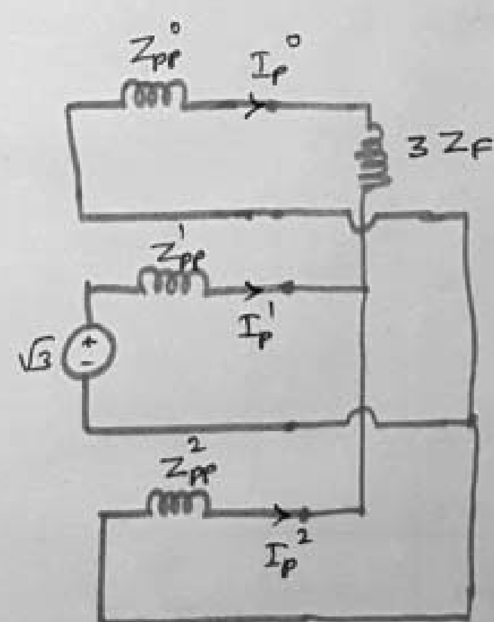
The fault current at faulted bus can be written as

$$[I_f^{012}] = \begin{bmatrix} I_p^0 \\ I_p^1 \\ I_p^2 \end{bmatrix} = \frac{\sqrt{3}}{Z_{pp}^1 + Z_{pp}^2 + Z_F} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad — \text{④}$$

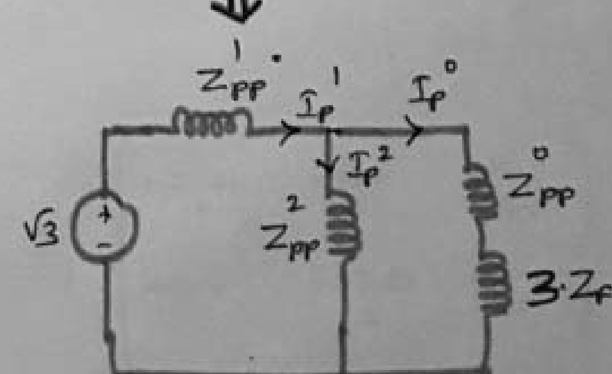
eqn ① and ③ may be used to calculate the bus voltages under fault condition.

## LLG fault :-




⇓

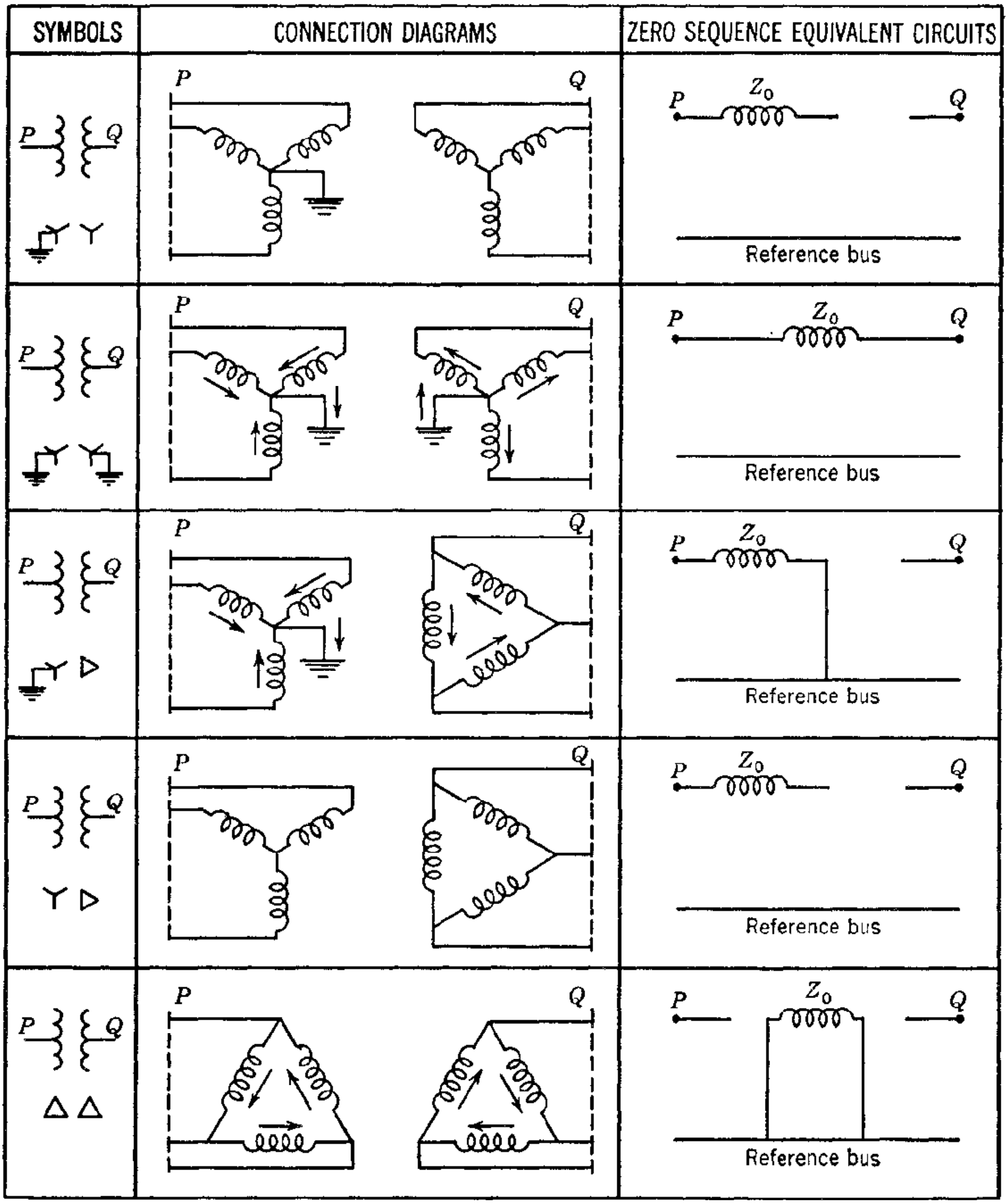



$$I_p^1 = \frac{\sqrt{3}}{Z_{pp}^1 + \left(Z_{pp}^2 \parallel Z_{pp}^0 + 3Z_F\right)}$$

$$I_p^2 = -I_p^1 * \frac{(Z_{pp}^0 + 3Z_F)}{Z_{pp}^2 + (Z_{pp}^0 + 3Z_F)}$$

$$I_p^0 = -I_p^1 * \frac{Z_{pp}^2}{Z_{pp}^2 + (Z_{pp}^0 + 3Z_F)}.$$

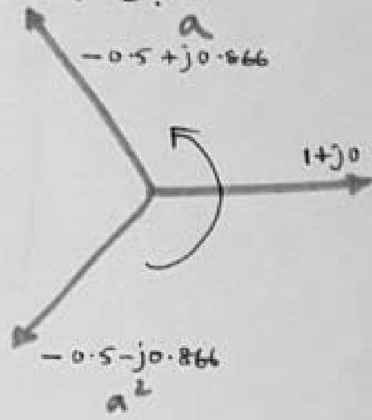
# Zero Sequence Equivalent Circuits of Three-Phase Transformers



## Conversion of Sequence quantities into Phase quantities.

The sequence components can be converted ~~by~~ into phase components by using Transformation matrix. $T_s$.

$$T_s = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$




$$\left[\underline{I}^{abc}\right] = \left[T_s\right]\left[\underline{I}^{012}\right]$$

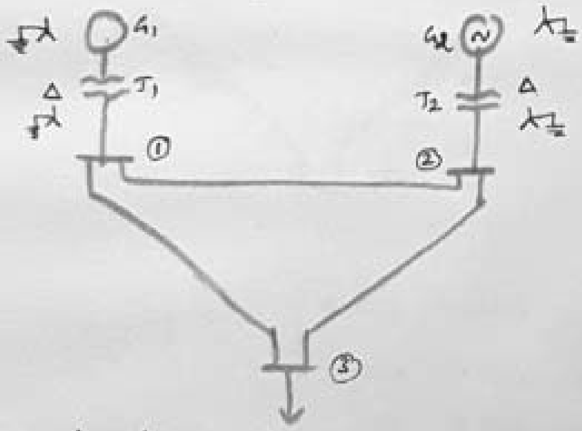
$$\left[V^{abc}\right] = \left[T_s\right]\left[V^{012}\right].$$

**Problem :-** For the system shown in fig, calculate the fault current, and bus voltages for the following faults.

ⓐ Single line to ground fault

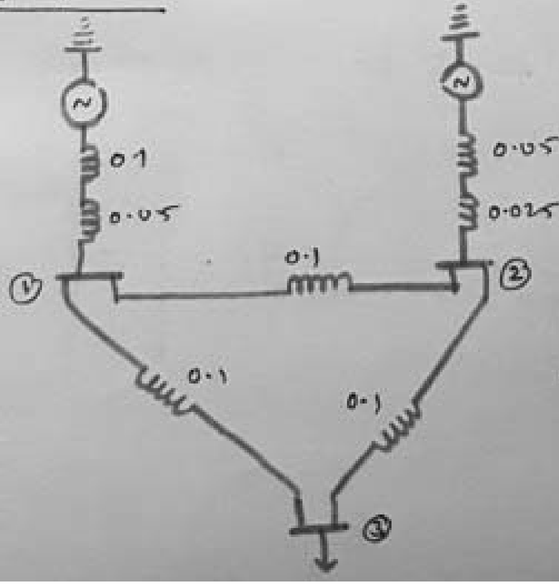
ⓑ ~~Double~~ Line to Line fault.

Assume the fault Impedance $Z_F = 0$. and fault occurs at bus 3.




| | $X^0$ | $X^1$ | $X^2$ |
|---|---|---|---|
| G1 | 0.05 | 0.1 | 0.1 |
| G2 | 0.025 | 0.05 | 0.05 |
| T1 | 0.05 | 0.05 | 0.05 |
| T2 | 0.025 | 0.025 | 0.025 |
| All lines | 0.2 | 0.1 | 0.1 |

+ve seq-network




$$Y^{+} = \begin{bmatrix} \underset{A}{+26.66} & \underset{B}{-10.0} & \underset{C}{-10.0} \\ \underset{D}{-10.0} & \underset{E}{+33.33} & \underset{F}{-10.0} \\ \underset{X}{-10.0} & \underset{Y}{-10.0} & \underset{M}{+20.0} \end{bmatrix}$$

$$\text{cofactor} = \begin{bmatrix} \overset{EMYF}{566.66} & \overset{DMXF}{300} & \overset{DYXE}{433.3} \\ \overset{BMYC}{300} & \overset{AMXC}{433.2} & \overset{AYXB}{366.6} \\ \overset{BFEC}{433.3} & \overset{AFDC}{366.6} & \overset{AEDB}{788.58} \end{bmatrix} \qquad |\Delta| = 7772.6$$

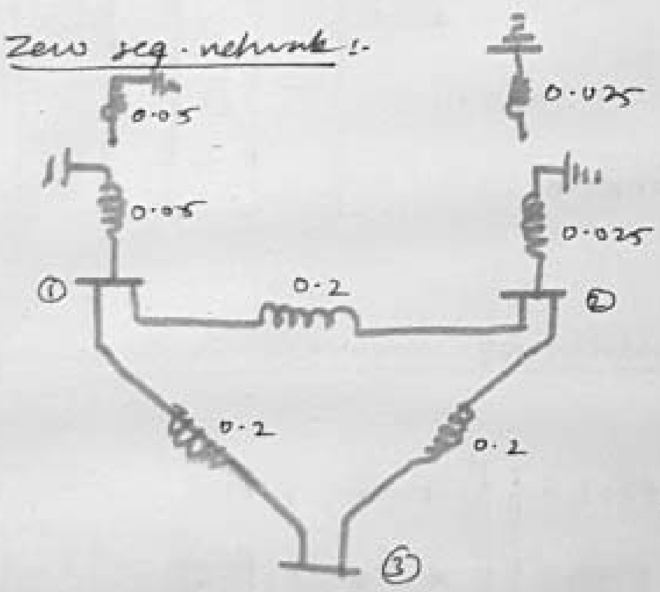
$$[Z^+] = \begin{bmatrix} 0.0729 & 0.0386 & 0.0557 \\ 0.0386 & 0.0557 & 0.0472 \\ 0.0557 & 0.0472 & 0.1015 \end{bmatrix}$$

**-ve seq. network.**

The -ve seq. network will be same as that of +ve seq. network without any source. The source is shorted and hence

$$[Z^-] = [Z^+].$$

**Zero seq. network :-**




$$Y^0 = \begin{bmatrix} 30 & -5 & -5 \\ -5 & 50 & -5 \\ -5 & -5 & 10 \end{bmatrix}$$

$$\text{cofactor} = \begin{bmatrix} 475 & 75 & 275 \\ 75 & 275 & 175 \\ 275 & 175 & 1475 \end{bmatrix}$$

$$|\Delta| = 12500$$

$$[Z^0] = \begin{bmatrix} 0.038 & 0.006 & 0.022 \\ 0.006 & 0.022 & 0.014 \\ 0.022 & 0.014 & 0.1180 \end{bmatrix}$$

**LG fault :-**

The fault current at fault bus ③ can be calculated using eqn. ②

$$\begin{bmatrix} I_3^0 \\ I_3^1 \\ I_3^2 \end{bmatrix} = \frac{\sqrt{3}}{0.1180 + 0.1015 + 0.1015} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5.3959 \\ 5.3959 \\ 5.3959 \end{bmatrix}$$

The fault bus voltage (bus 3) can be calculated using eqn (1)

$$\begin{bmatrix} V_3^0 \\ V_3^1 \\ V_3^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} 0.1180 & & \\ & 0.1015 & \\ & & 0.1015 \end{bmatrix} \begin{bmatrix} 5.3959 \\ 5.3959 \\ 5.3959 \end{bmatrix} = \begin{bmatrix} -0.6367 \\ 1.1844 \\ -0.5477 \end{bmatrix}$$

The voltages at all other buses can be calculated using eqn (3)

$$\begin{bmatrix} V_1^0 \\ V_1^1 \\ V_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} 0.022 & & \\ & 0.0557 & \\ & & 0.0557 \end{bmatrix} \begin{bmatrix} 5.3959 \\ 5.3959 \\ 5.3959 \end{bmatrix} = \begin{bmatrix} -0.1187 \\ 1.4315 \\ -0.3006 \end{bmatrix}$$

$$\begin{bmatrix} V_2^0 \\ V_2^1 \\ V_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} 0.014 & & \\ & 0.0472 & \\ & & 0.0472 \end{bmatrix} \begin{bmatrix} 5.3959 \\ 5.3959 \\ 5.3959 \end{bmatrix} = \begin{bmatrix} -0.0755 \\ 1.4774 \\ -0.2547 \end{bmatrix}$$

Conversion of Sequence quantities into Phase quantities.

$a^2 = -0.5 - j0.866$
$a = -0.5 + j0.866$

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 5.3959 \\ 5.3959 \\ 5.3959 \end{bmatrix} = \begin{bmatrix} 9.3459 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.3459 \\ 0 \\ 0 \end{bmatrix}.$$

$$V_1^{abc} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.1187 \\ 1.4315 \\ -0.3006 \end{bmatrix} = \begin{bmatrix} 0.5844 \\ -0.395 - j0.866 \\ -0.395 + j0.866 \end{bmatrix} = \begin{bmatrix} 0.5844\angle 0^\circ \\ 0.9518\angle -114.52 \\ 0.9518\angle +114.52 \end{bmatrix}$$

$$V_2^{abc} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0755 \\ 1.4774 \\ -0.2547 \end{bmatrix} = \begin{bmatrix} 1.1472 \\ -0.3966 - j0.866 \\ -0.3966 + j0.866 \end{bmatrix} = \begin{bmatrix} 1.1472\angle 0^\circ \\ 0.9524\angle -114.6 \\ 0.9525\angle +114.6 \end{bmatrix}$$

$$V_3^{abc} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.6367 \\ 1.1844 \\ -0.5477 \end{bmatrix} = \begin{bmatrix} 0.0 \\ -0.5514 - j0.866 \\ -0.5514 + j0.866 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.0266\angle -122.5 \\ 1.0266\angle 122.5 \end{bmatrix}$$

LL fault at bus 3 :-

$$\left[I_3^{012}\right] = \frac{\sqrt{3}}{0.1015 + 0.1015}\begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}0.0\\8.5323\\-8.5323\end{bmatrix}$$ → fault bus current.

$$\left[V_1^{012}\right] = \begin{bmatrix}0\\\sqrt{3}\\0\end{bmatrix} - \begin{bmatrix}0.022 & & \\ & 0.0557 & \\ & & 0.0557\end{bmatrix}\begin{bmatrix}0.0\\8.5323\\-8.5323\end{bmatrix} = \begin{bmatrix}0.0\\1.2457\\0.4863\end{bmatrix}$$ other bus voltage

$$\left[V_2^{012}\right] = \begin{bmatrix}0\\\sqrt{3}\\0\end{bmatrix} - \begin{bmatrix}0.014 & & \\ & 0.0472 & \\ & & 0.0472\end{bmatrix}\begin{bmatrix}0.0\\8.5323\\-8.5323\end{bmatrix} = \begin{bmatrix}0.0\\1.3293\\0.4027\end{bmatrix}$$ other bus voltage

$$\left[V_3^{012}\right] = \begin{bmatrix}0\\\sqrt{3}\\0\end{bmatrix} - \begin{bmatrix}0.1180 & & \\ & 0.1015 & \\ & & 0.1015\end{bmatrix}\begin{bmatrix}0.0\\8.5323\\-8.5321\end{bmatrix} = \begin{bmatrix}0\\0.8660\\0.8660\end{bmatrix}$$ fault bus voltage.

Conversion into phase quantities:

$$I_3^{abc} = [T_s]\begin{bmatrix}0\\8.5323\\-8.5323\end{bmatrix} = \begin{bmatrix}0.0\\8.5323\\-8.5323\end{bmatrix} = \begin{bmatrix}0.0\\-j8.5323\\+j8.5323\end{bmatrix}$$ fault bus current

$$V_1^{abc} = [T_s]\begin{bmatrix}0\\1.2457\\0.4863\end{bmatrix} = \begin{bmatrix}1.0\\-0.5-j0.3797\\-0.5+j0.3799\end{bmatrix}$$ other bus voltage

$$V_2^{abc} = [T_s]\begin{bmatrix}0.0\\1.3293\\0.4027\end{bmatrix} = \begin{bmatrix}1.0\\-0.5-j0.4633\\-0.5+j0.4633\end{bmatrix}$$ other bus voltage

$$V_3^{abc} = [T_s]\begin{bmatrix}0.0\\0.866\\0.866\end{bmatrix} = \begin{bmatrix}1.0\\-0.5+j0\\-0.5+j0\end{bmatrix}$$ fault bus voltage.

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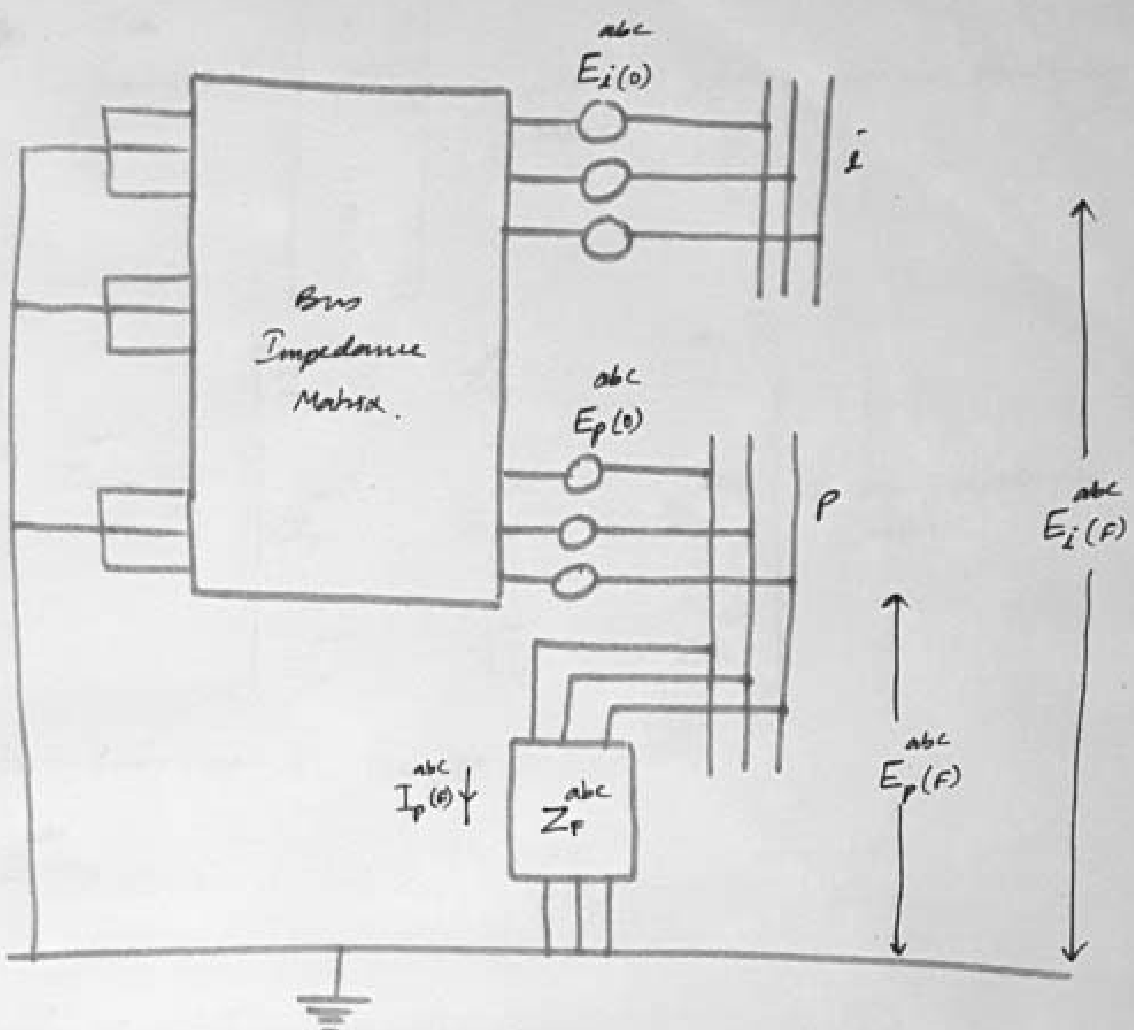
## Short Circuit Studies by Matrix method :-




The representation of the system with a fault at bus 'p' is shown in fig. In this representation, derived by means of Thevenin's theorem, the internal impedance is represented by the bus impedance matrix including machine reactance and the open circuited voltage is represented by the bus voltage prior to the fault.

The performance equation of the system during fault is

$$E_{bus(F)}^{abc} = E_{bus(0)}^{abc} - Z_{bus}^{abc} \cdot I_{bus(F)}^{abc} \quad \text{2(1)}$$

where

$$E_{bus(F)}^{abc} = \begin{bmatrix} E_1^{abc}(F) \\ \vdots \\ E_p^{abc}(F) \\ \vdots \\ E_n^{abc}(F) \end{bmatrix} = \text{bus voltages during fault (unknown voltage vector)}$$

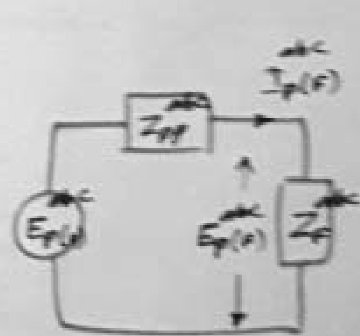
$$E_{bus(0)}^{abc} = \begin{bmatrix} E_1^{abc}(0) \\ \vdots \\ E_p^{abc}(0) \\ \vdots \\ E_n^{abc}(0) \end{bmatrix} = \text{voltage vector prior to fault.}$$

$$I^{abc}_{bus(F)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I^{abc}_{p(F)} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \text{current vector during fault at bus } p$$

$$Z^{abc}_{bus} = \begin{bmatrix} Z^{abc}_{11} & \cdots & Z^{abc}_{1p} & \cdots & Z^{abc}_{1n} \\ \vdots & & \vdots & & \\ Z^{abc}_{p1} & \cdots & Z^{abc}_{pp} & \cdots & Z^{abc}_{pn} \\ \vdots & & \vdots & & \\ Z^{abc}_{n1} & \cdots & Z^{abc}_{np} & \cdots & Z^{abc}_{nn} \end{bmatrix} = \text{bus impedance matrix.}$$

Expanding eqn (1), we get

$$\begin{aligned} E^{abc}_{1(F)} &= E^{abc}_{1(0)} - Z^{abc}_{1p} \cdot I^{abc}_{p(F)} \\ &\vdots \\ E^{abc}_{p(F)} &= E^{abc}_{p(0)} - Z^{abc}_{pp} \cdot I^{abc}_{p(F)} \\ &\vdots \\ E^{abc}_{n(F)} &= E^{abc}_{n(0)} - Z^{abc}_{np} \cdot I^{abc}_{p(F)} \end{aligned} \qquad 2\ (2)$$




The fault voltage across the fault impedance can be written as

$$\boxed{E^{abc}_{p(F)} = Z^{abc}_{F} \cdot I^{abc}_{p(F)}} \qquad 2\ (3)$$

Equate eqn (3) with $p^{th}$ eqn of (2), we get

$$Z^{abc}_{F} \cdot I^{abc}_{p(F)} = E^{abc}_{p(0)} - Z^{abc}_{pp} \cdot I^{abc}_{p(F)}$$

$$\boxed{I^{abc}_{p(F)} = \left(Z^{abc}_{pp} + Z^{abc}_{F}\right)^{-1} \cdot E^{abc}_{p(0)}} \qquad 2\ (4)$$

Steps:

The fault bus current can be calculated using eq (4).
The fault bus voltage can be calculated using eq (3)
The other bus voltages can be calculated using eq (2)

Fault in ~~...~~ admittance form :- When it is desirable to express the fault in admittance form, the three phase fault current at bus p can be written as

$$\boxed{I_{p(F)}^{abc} = Y_F^{abc} \cdot E_{p(F)}^{abc}} \quad \text{— (5)}$$

Sub. (5) in $p^{th}$ eqn of (2)

$$E_{p(F)}^{abc} = E_{p(0)}^{abc} - Z_{pp}^{abc} \cdot Y_F^{abc} \cdot E_{p(F)}^{abc}$$

$$\boxed{E_{p(F)}^{abc} = \left( U + Z_{pp}^{abc} \cdot Y_F^{abc} \right)^{-1} \cdot E_{p(0)}^{abc}} \quad \text{— (6)}$$

Steps :-

The fault bus voltage can be calculated using eqn (6)
The fault bus current can be calculated using eqn (5)
The other bus voltages during fault can be calculated using eqn (2)

Transformation to symmetrical components :-

The formulas developed in the preceding section can be simplified by using symmetrical components. The primitive impedance matrix for a $3\phi$ element is

$$Z_{pq}^{abc} = \begin{bmatrix} Z_{pq}^{s} & Z_{pq}^{m} & Z_{pq}^{m} \\ Z_{pq}^{m} & Z_{pq}^{s} & Z_{pq}^{m} \\ Z_{pq}^{m} & Z_{pq}^{m} & Z_{pq}^{s} \end{bmatrix}.$$

The matrix can be diagonalised by the transformation $(T_s^{*})^t Z_{pq}^{abc} T_s$ into

$$Z_{pq}^{012} = \begin{bmatrix} Z_{pq}^{0} & & \\ & Z_{pq}^{1} & \\ & & Z_{pq}^{2} \end{bmatrix}$$ ; where $Z_{pq}^{0}, Z_{pq}^{1}\ Z_{pq}^{2}$ are zero, positive and negative sequence impedance

The $p^{th}$ bus voltage prior to fault is

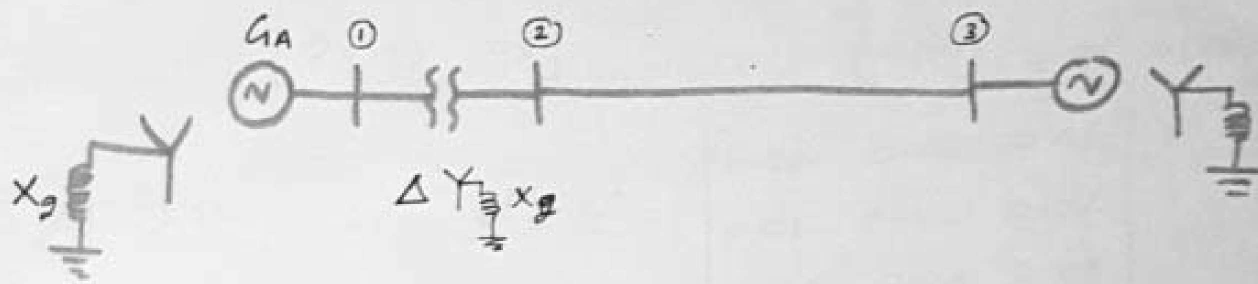
$$E_{i(0)}^{abc} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$ ; Transforming into symmetrical components, that is,

$$E_{i(0)}^{012} = (T_s^{*})^t E_{i(0)}^{abc}$$

$$E_{i(0)}^{012} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}.$$

The fault impedance matrix $Z_F^{abc}$ can be transformed by $T_s$ into the matrix $Z_F^{012}$. The resulting matrix is diagonal if the fault is balanced.

The equations ① – ⑥ can be suitably modified by replacing the superscripts abc by 012. The fault impedance and admittance matrices in terms of three phase and symmetrical components for various faults are given in Table.




The fig. shows the one-line diagram of power system. Impedance data are as follows.

For gen $G_A$ & $G_B$ ; $X_1 = X_2 = 0.1$ ; $X_0 = 0.04$ and $X_g = 0.02$

For transformer ; $X_1 = X_2 = 0.1$ ; $X_0 = 0.1$ and $X_g = 0.05$

The $3\phi$ reactance matrix for the line is

$$X_{line} = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0.3 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \end{array}$$

The system is initially in balanced operation and may be considered to be unloaded. Find all the voltages and currents when a L-G fault with fault impedance of $j0.005$ occurs at bus 2.

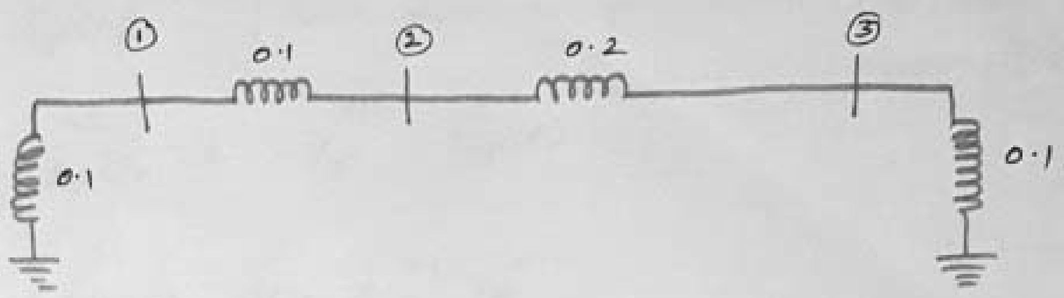
Soln :

From the reactance matrix of the transmission line, the sequence components can be computed as follows.

$$\underline{x^1 = x^2} = X_{self} - X_{mutual} = 0.3 - 0.1 = \underline{0.2}$$

$$\underline{x^0} = X_{self} + 2 * X_{mutual} = 0.3 + (2 * 0.1) = \underline{0.5}$$

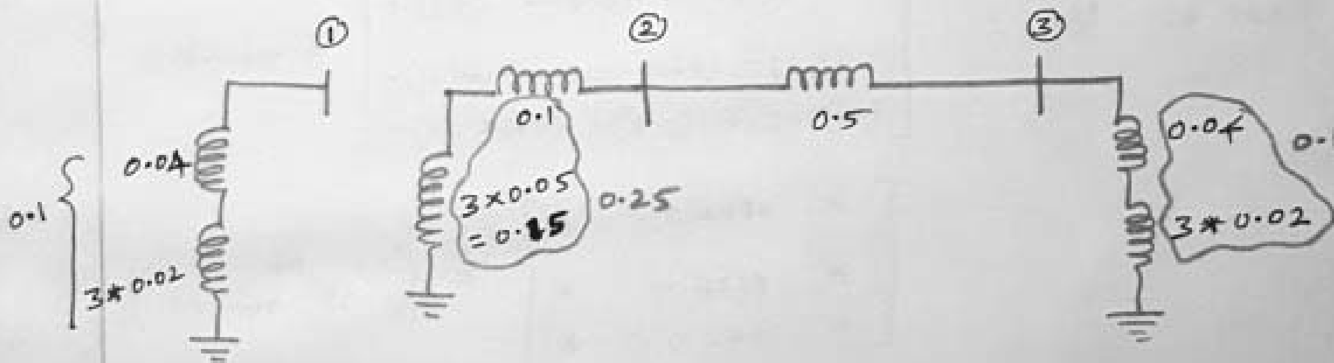
+ve and -ve seq. network.




$$Y^1 = Y^2 = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 15 & -5 \\ 0 & -5 & 15 \end{bmatrix} \qquad \text{cofactor} = \begin{bmatrix} +(200) & -(-150) & +(50) \\ -(-150) & +(300) & -(-100) \\ +(50) & -(-100) & +(200) \end{bmatrix}$$

$$|\Delta| = 2500 \quad ; \quad Z^1 = Z^2 = \begin{bmatrix} 0.08 & 0.06 & 0.02 \\ 0.06 & 0.12 & 0.04 \\ 0.02 & 0.04 & 0.08 \end{bmatrix}$$

zero seq. network :-

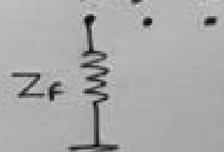



$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ Y^0 = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 10 & 0 & 0 \\ 0 & 6.0 & -2.0 \\ 0 & -2.0 & 12 \end{bmatrix} \end{array} \qquad \begin{bmatrix} 6.0 & -2.0 \\ -2.0 & 12 \end{bmatrix}^{-1} = \begin{bmatrix} 0.1765 & 0.0294 \\ 0.0294 & 0.0882 \end{bmatrix}$$

$$|\Delta| = 68$$

$$Z^0 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1765 & 0.0294 \\ 0 & 0.0294 & 0.0882 \end{bmatrix}$$

Fault admittance matrix




$$\frac{Y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ; \quad Y_F = \frac{1}{Z_F} = \frac{1}{0.005} = 200.0$$

$$Y_F/3 = 66.667$$

$$Y_F^{012} = \begin{bmatrix} 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \end{bmatrix}$$

$$E_p^{012}(F) = \left(U + Z_{pp}^{012} Y_F^{012}\right)^{-1} E_p^{012}(0)$$

$$I_p^{012}(F) = Y_F^{012} \cdot E_p^{012}(F)$$

pl. refer eqn ⑤ & ⑥

Calculation of $U + Z_{pp}^{012} Y_F^{012}$

$$= \begin{bmatrix} 1.0 & & \\ & 1.0 & \\ & & 1.0 \end{bmatrix} + \begin{bmatrix} 0.1765 & & \\ & 0.12 & \\ & & 0.12 \end{bmatrix} \begin{bmatrix} 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0 & & \\ & 1.0 & \\ & & 1.0 \end{bmatrix} + \begin{bmatrix} 12.7667 & 11.7667 & 11.7667 \\ 8.0 & 8.0 & 8.0 \\ 8.0 & 8.0 & 8.0 \end{bmatrix} = \begin{bmatrix} 12.7667 & 11.7667 & 11.7667 \\ 8.0 & 9.0 & 8.0 \\ 8.0 & 8.0 & 9.0 \end{bmatrix}$$

Calculation of $\left(U + Z_{pp}^{012} Y_F^{012}\right)^{-1}$

$$\text{cofactor} = \begin{bmatrix} +(17) & -(8.0) & +(-8.0) \\ -(11.7667) & +(20.7667) & -(8.0) \\ +(-11.7667) & -(8.0) & +(20.7667) \end{bmatrix} \quad ; \quad |\Delta| = 28.7667$$

$$\left(U + Z_{pp}^{012} Y_F^{012}\right)^{-1} = \begin{bmatrix} * & -0.4090 & * \\ * & 0.7219 & * \\ * & -0.2781 & * \end{bmatrix}$$

eqn ⑥ $$E_2^{012}(F) = \begin{bmatrix} * & -0.4090 & * \\ * & 0.7219 & * \\ * & -0.2781 & * \end{bmatrix} \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.7084 \\ 1.2504 \\ -0.4817 \end{bmatrix}$$

eqn ⑤ $$I_2^{012}(F) = \begin{bmatrix} 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \\ 66.667 & 66.667 & 66.667 \end{bmatrix} \begin{bmatrix} -0.7084 \\ 1.2504 \\ -0.4817 \end{bmatrix} = \begin{bmatrix} 4.0211 \\ 4.0211 \\ 4.0211 \end{bmatrix}$$

Using eqn ②

$$E_1^{012}(F) = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} 0.0 & & \\ & 0.06 & \\ & & 0.06 \end{bmatrix} \begin{bmatrix} 4.0211 \\ 4.0211 \\ 4.0211 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.4908 \\ -0.2413 \end{bmatrix}$$

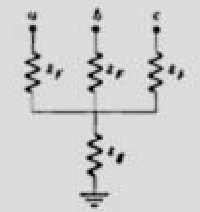
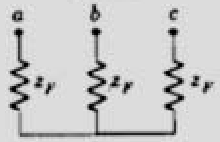
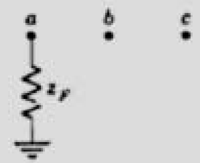
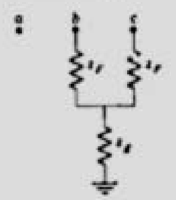
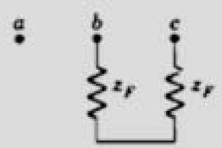
$$E_3^{012}(F) = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} - \begin{bmatrix} 0.0294 & & \\ & 0.04 & \\ & & 0.04 \end{bmatrix} \begin{bmatrix} 4.0211 \\ 4.0211 \\ 4.0211 \end{bmatrix} = \begin{bmatrix} -0.1182 \\ 1.5712 \\ -0.1608 \end{bmatrix}.$$

Calculation of currents thro' transformer (from bus 1 to 2)

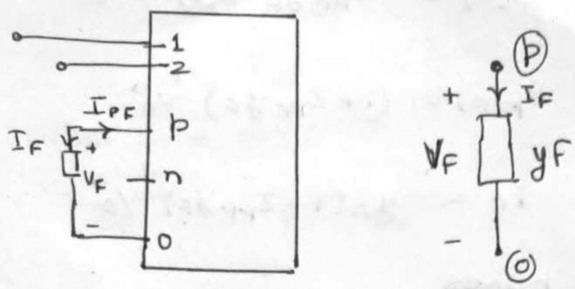
$$I_{12}^{012} = \begin{bmatrix} \dfrac{0-(-0.7084)}{0.25} \\[2ex] \dfrac{1.4908-1.2504}{0.1} \\[2ex] \dfrac{(-0.2413)-(-0.4817)}{0.1} \end{bmatrix} = \begin{bmatrix} 2.8336 \\[2ex] 2.4040 \\[2ex] 2.4040 \end{bmatrix}$$

line reactance

# FAULT IMPEDANCE AND ADMITTANCE MATRICES

| Type of fault | Three-phase components | | Symmetrical components | |
|---|---|---|---|---|
| | $Z_F^{a,b,c}$ | $Y_F^{a,b,c}$ | $Z_F^{0,1,2}$ | $Y_F^{0,1,2}$ |
|  Three-phase-to-ground | $\begin{bmatrix} z_F+z_g & z_g & z_g \\ z_g & z_F+z_g & z_g \\ z_g & z_g & z_F+z_g \end{bmatrix}$ | $\frac{1}{3}\begin{bmatrix} y_0+2y_F & y_0-y_F & y_0-y_F \\ y_0-y_F & y_0+2y_F & y_0-y_F \\ y_0-y_F & y_0-y_F & y_0+2y_F \end{bmatrix}$ where $y_0 = \frac{1}{z_F+3z_g}$ | $\begin{bmatrix} z_F+3z_g & 0 & 0 \\ 0 & z_F & 0 \\ 0 & 0 & z_F \end{bmatrix}$ | $\begin{bmatrix} y_0 & 0 & 0 \\ 0 & y_F & 0 \\ 0 & 0 & y_F \end{bmatrix}$ where $y_0 = \frac{1}{z_F+3z_g}$ |
|  Three-phase | Not defined | $\frac{y_F}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ | $\begin{bmatrix} \infty & 0 & 0 \\ 0 & z_F & 0 \\ 0 & 0 & z_F \end{bmatrix}$ | $y_F\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ |
|  Line-to-ground | $\begin{bmatrix} z_F & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix}$ | $\begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ | Not defined | $\frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ |
|  Line-to-line-to-ground | $\begin{bmatrix} \infty & 0 & 0 \\ 0 & z_F+z_g & z_g \\ 0 & z_g & z_F+z_g \end{bmatrix}$ | $\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{z_F+z_g}{z_F^2+2z_Fz_g} & \frac{-z_g}{z_F^2+2z_Fz_g} \\ 0 & \frac{-z_g}{z_F^2+2z_Fz_g} & \frac{z_F+z_g}{z_F^2+2z_Fz_g} \end{bmatrix}$ | Not defined | $\frac{1}{3(z_F^2+2z_Fz_g)}\begin{bmatrix} 2z_F & -z_F & -z_F \\ -z_F & 2z_F+3z_g & -(z_F+3z_g) \\ -z_F & -(z_F+3z_g) & 2z_F+3z_g \end{bmatrix}$ |
|  Line-to-line | Not defined | $\frac{y_F}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ | Not defined | $\frac{y_F}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ |

## Symetrical Three Phase to Ground Fault



Suppose the fault occurs at 'p'th bus. If the fault is in admittance form, the fault description is given by

$$I_F = y_F V_F \qquad — (1)$$

The pth port is terminated with the admittance $y_F$. The constraints between the variables of the fault admittance and port variables are written as

$$\left.\begin{aligned} &I_F = -I_{p(F)} \quad V_F = V_{p(F)} \\ &I_{i(F)} = 0 \;; \quad V_{i(F)} = \text{unknown} \\ &\qquad\qquad \text{where } i = 1, 2, \ldots n \\ &\qquad\qquad\qquad i \neq p \end{aligned}\right\} — (2)$$

where $I_{p(F)}$ and $V_{p(F)}$ are the port current and port voltage variables, under faulted condition.

$$\begin{aligned} V_{1(F)} &= Z_{11} I_{1(F)} + \ldots + Z_{1p} I_{p(F)} + \ldots + Z_{1n} I_{n(F)} + V_0^a \\ V_{2(F)} &= Z_{21} I_{1(F)} + \ldots + Z_{2p} I_{p(F)} + \ldots + Z_{2n} I_{n(F)} + V_0^a \\ &\vdots \\ V_{p(F)} &= Z_{p1} I_{1(F)} + \ldots + Z_{pp} I_{p(F)} + \ldots + Z_{pn} I_{n(F)} + V_0^a \\ &\vdots \\ V_{n(F)} &= Z_{n1} I_{1(F)} + \ldots + Z_{np} I_{p(F)} + \ldots + Z_{nn} I_{n(F)} + V_0^a \end{aligned} \qquad (3)$$

From pth equation in (3) and using relations (1) and (2) we can write

$$V_{p(F)} = -Z_{pp} y_F V_{p(F)} + V_0^a \qquad — (4)$$

Hence

$$V_{p(F)} = (1 + Z_{pp} y_F)^{-1} V_0^a \qquad — (5)$$

$$I_F = y_F (1 + Z_{pp} y_F)^{-1} V_0^a \qquad — (6)$$

At other buses

$$V_{iF} = V_0^a + Z_{ip} I_{p(F)}$$

$$= V_0^a - Z_{ip} I_F$$

$$= V_0^a - Z_{ip} y_F (1+Z_{pp} y_F)^{-1} V_0^a \quad -⑦$$

Thus all bus voltages (Unknown) are determined.

**Symmetrical Three-phase fault - not involving ground**

We have only the positive sequence admittance

$$I_F = y_F V_F$$

The results given by equations ⑤, ⑥ & ⑦ are applicable.

**Fault analysis in Phase Impedance form**

In three-phase form

$$[V_{bus}^{a,b,c}] = [Z_{bus}^{abc}][I_{bus}^{abc}] + [B][V_0^{abc}] \quad -①$$

$$\text{where } [B] = \begin{bmatrix} U \\ U \\ \vdots \\ U \end{bmatrix} \quad V_0^{abc} = \begin{bmatrix} V_0^a \\ V_0^b \\ V_0^c \end{bmatrix}$$

**Fault in admittance form**

$$I_F^{abc} = Y_F^{abc} V_F^{abc} \quad -②$$

For a fault at $p^{th}$ bus the fault currents and voltages are

$$I_F^{abc} = Y_F^{abc} [U + Z_{pp}^{abc} Y_F^{abc}]^{-1} V_0^{abc} \quad -③$$

$$\left.\begin{array}{l} I_F^{abc} = -I_{p(F)}^{abc} \\ I_{i(F)}^{abc} = 0 \quad \text{for } i = 1,2,..n \\ \qquad\qquad\qquad i \neq p \end{array}\right\} \quad -④$$

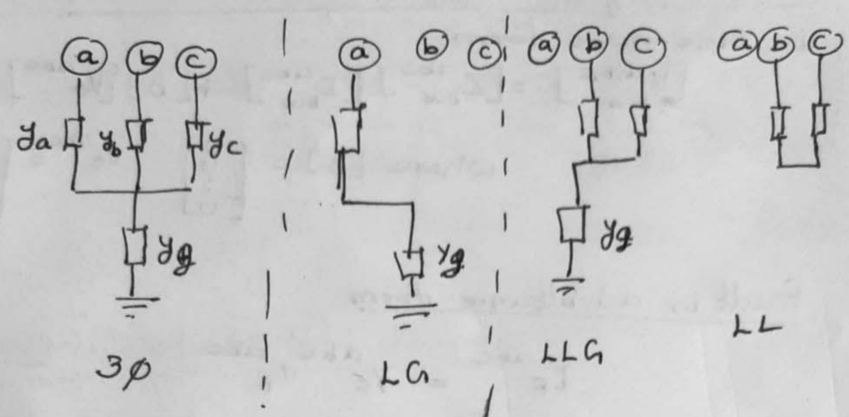
Voltages are given by

$$V_{i(F)}^{abc} = V_0^{abc} - Z_{ip}^{abc} I_F^{abc} \quad \text{for } i = 1,2 \ldots n, \; i \neq p \quad -⑤$$

$$V_{p(F)}^{abc} = [U + Z_{pp}^{abc} Y_F^{abc}]^{-1} V_0^{abc} \quad -⑥$$

For various types of unsymmetrical faults, the appropriate $Y_F^{abc}$ is substituted in the above equation $Y_F^{abc}$.

## Fault Representation in Phase Quantities




In admittance form ($3\phi$ symmetrical fault)

$$\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix} = \frac{1}{y}\begin{bmatrix} y_a(y_b+y_c+y_g) & -y_a y_b & -y_a y_c \\ -y_a y_b & y_b(y_a+y_c+y_g) & -y_b y_c \\ -y_a y_c & -y_b y_c & y_c(y_a+y_b+y_g) \end{bmatrix}\begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix} \quad \text{①}$$

i.e $[I_F^{abc}] = [Y_F^{abc}][V_F^{abc}]$

$$y = y_a + y_b + y_c + y_g$$

### Single Line to Ground Fault (LG)

$$y_b = y_c = 0$$

from ①

$$Y_F^{abc} = \begin{bmatrix} \frac{y_a y_g}{y_a + y_g} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{②}$$

### Line-Line to Ground Fault (LLG) $y_a = 0$

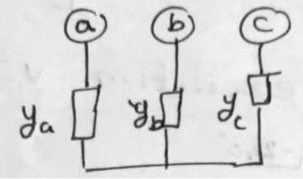
From ①

$$Y_F^{abc} = \frac{1}{y}\begin{bmatrix} 0 & 0 & 0 \\ 0 & y_b(y_c+y_g) & -y_b y_c \\ 0 & -y_b y_c & y_c(y_b+y_g) \end{bmatrix} \quad \text{③}$$

### Line-Line Fault (LL) $y_a = 0$ $y_g = 0$

$$Y_F^{abc} = \frac{1}{y_b + y_c}\begin{bmatrix} 0 & 0 & 0 \\ 0 & y_b y_c & -y_b y_c \\ 0 & -y_b y_c & + y_b y_c \end{bmatrix} \quad \text{④}$$

### Three-Phase Fault Unsymmetrical



$y_g = 0$

$$\therefore \quad Y_F^{abc} = \frac{1}{y}\begin{bmatrix} y_a(y_b+y_c) & -y_a y_b & -y_a y_c \\ -y_a y_b & y_b(y_a+y_c) & -y_b y_c \\ -y_a y_c & -y_b y_c & y_c(y_a+y_b) \end{bmatrix} \quad \text{--- (5)}$$

Using equations (2) to (5) the fault currents and voltages for any unsymmetrical fault can be found out.

## Expressions for Voltages and Currents under Faulted Condition – Symmetrical Comp. Analysis

The voltages behind transient reactances are expressed as

$$V^S = V_0^{012} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_0^a \\ V_0^b \\ V_0^c \end{bmatrix}$$

For balanced excitation $V_0^a = |V_a| \angle 0$ $V_0^b = |V_a| \angle -120^\circ$ and $V_0^c = |V_a| \angle -240^\circ$

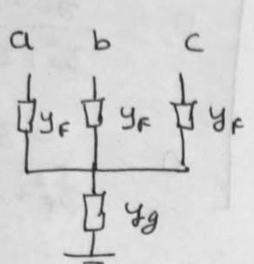
Hence

$$V^S = V^{012} = \begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix} |V_a|$$

The fault description in admittance form $Y_F^{012}$ for various types of faults are obtained by applying the symmetrical component transformation to the corresponding $Y_F^{abc}$

The fault admittance matrices ~~for~~ in the phase component form are summarized below.

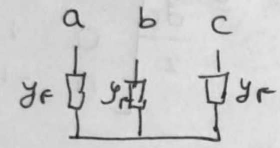
(1) $3\phi$ Symmetrical fault




$$Y_F^{abc} = \frac{1}{3}\begin{bmatrix} y_1+2y_F & y_1-y_F & y_1-y_F \\ y_1-y_F & y_1+2y_F & y_1-y_F \\ y_1-y_F & y_1-y_F & y_1+2y_F \end{bmatrix}$$

where $y_1 = \dfrac{1}{z_F + 3z_g}$

② $3\phi$ Unsymmetrical Fault

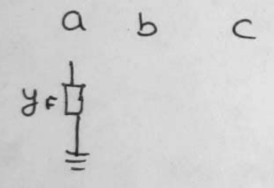



$$Y_F^{abc} = \frac{y_F}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

i.e $y_1 = 0$ in the above case

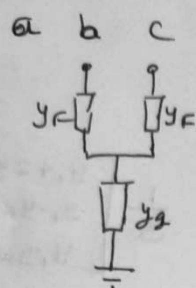
$\because z_g = \infty$

③ LG



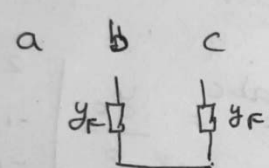

$$Y_F^{abc} = \begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

④ LLG




$$Y_F^{abc} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{z_F + z_g}{z_F^2 + 2z_F z_g} & \frac{-z_g}{z_F^2 + 2z_F z_g} \\ 0 & \frac{-z_g}{z_F^2 + 2z_F z_g} & \frac{z_F + z_g}{z_F^2 + 2z_F z_g} \end{bmatrix}$$

⑤ LL




$$Y_F^{abc} = \frac{y_F}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

For LG Fault

By symmetrical component transformation

$$Y_F^{012} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} y_F & y_F & y_F \\ y_F & y_F & y_F \\ y_F & y_F & y_F \end{bmatrix}$$

$$= \frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

For LG Fault

By symmetrical component transformation

$$Y_F^{012} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} y_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} y_F & y_F & y_F \\ y_F & y_F & y_F \\ y_F & y_F & y_F \end{bmatrix}$$

$$= \frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$I_F^{012} = Y_F^{012}\left[U + Z_F^{012} Y_F^{012}\right]^{-1} V_0^{012}$$

$$= \frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\left\{\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{U} + \underbrace{\begin{bmatrix} Z_{pp}^{(0)} & 0 & 0 \\ 0 & Z_{pp}^{(1)} & 0 \\ 0 & 0 & Z_{pp}^{(2)} \end{bmatrix}}_{Z_F^{012}} \times \underbrace{\frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{Y_F^{012}}\right\}^{-1} \times \underbrace{\begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}|V_a|}_{V_0^{012}}$$

Simplifying the above

$$I_F^{012} = \frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\left\{\begin{bmatrix} 1+Z_{pp}^{(0)}\frac{y_F}{3} & Z_{pp}^{(0)}\frac{y_F}{3} & Z_{pp}^{(0)}\frac{y_F}{3} \\ Z_{pp}^{(1)}\frac{y_F}{3} & 1+Z_{pp}^{(1)}\frac{y_F}{3} & Z_{pp}^{(1)}\frac{y_F}{3} \\ Z_{pp}^{(2)}\frac{y_F}{3} & Z_{pp}^{(2)}\frac{y_F}{3} & 1+Z_{pp}^{(2)}\frac{y_F}{3} \end{bmatrix}\right\}^{-1}\begin{bmatrix} 0 \\ \sqrt{3} \\ 0 \end{bmatrix}|V_a|$$

$$= \frac{\sqrt{3}}{Z_{pp}^{(0)} + 3Z_F + Z_{pp}^{(1)} + Z_{pp}^{(2)}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}|V_a|$$

Similarly for symmetrical $3\phi$ fault

$$Y_F^{012} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Y_F^{abc} \end{bmatrix}\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} y_0 & 0 & 0 \\ 0 & y_F & 0 \\ 0 & 0 & y_F \end{bmatrix}$$ where $y_0 = \frac{1}{z_F + 3z_g}$

For $3\phi$ fault without ground

$$Y_F^{012} = y_F\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ $\because z_g = \infty$ in the above expression

For LG Fault

$$Y_F^{012} = \frac{y_F}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

For LLG

$$Y_F^{012} = \frac{1}{3(Z_F^2 + 2Z_f Z_g)} \begin{bmatrix} 2Z_F & -Z_F & -Z_F \\ -Z_F & 2Z_F + 3Z_g & -(Z_F + 3Z_g) \\ -Z_F & -(Z_f + 3Z_g) & 2Z_f + 3Z_g \end{bmatrix}$$

For LL

$$Y_F^{012} = \frac{y_F}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$